Ein Blick ----- Einblick

Wie wir in „Mathematik für alle“ die Welt der Mathematik sehen

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a sight in ----- an insight

How we see the world of mathematics in „mathematics für everybody“.

Ein Weg ist gangbar vorbereitet

Wie wir in „Mathematik für alle“ die Welt der Mathematik sehen

A Viable Path is Prepared.

How we see the world of mathematics in „mathematics für everybody“.

Exponentialfunktion

Exp-fkt

$$f(x) = k^x$$

$k > 0,\ Def = \mathbb{R}$

$k = 0,\ Def = \mathbb{R}^+$

Basis k >1

Basis k mit 0<k <1

für Basis k <0 ist f nicht definiert

Exponential Functions

Exp-fkt

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base k >1

basis k with 0<k <1

for basis k <0 there is no definition for f

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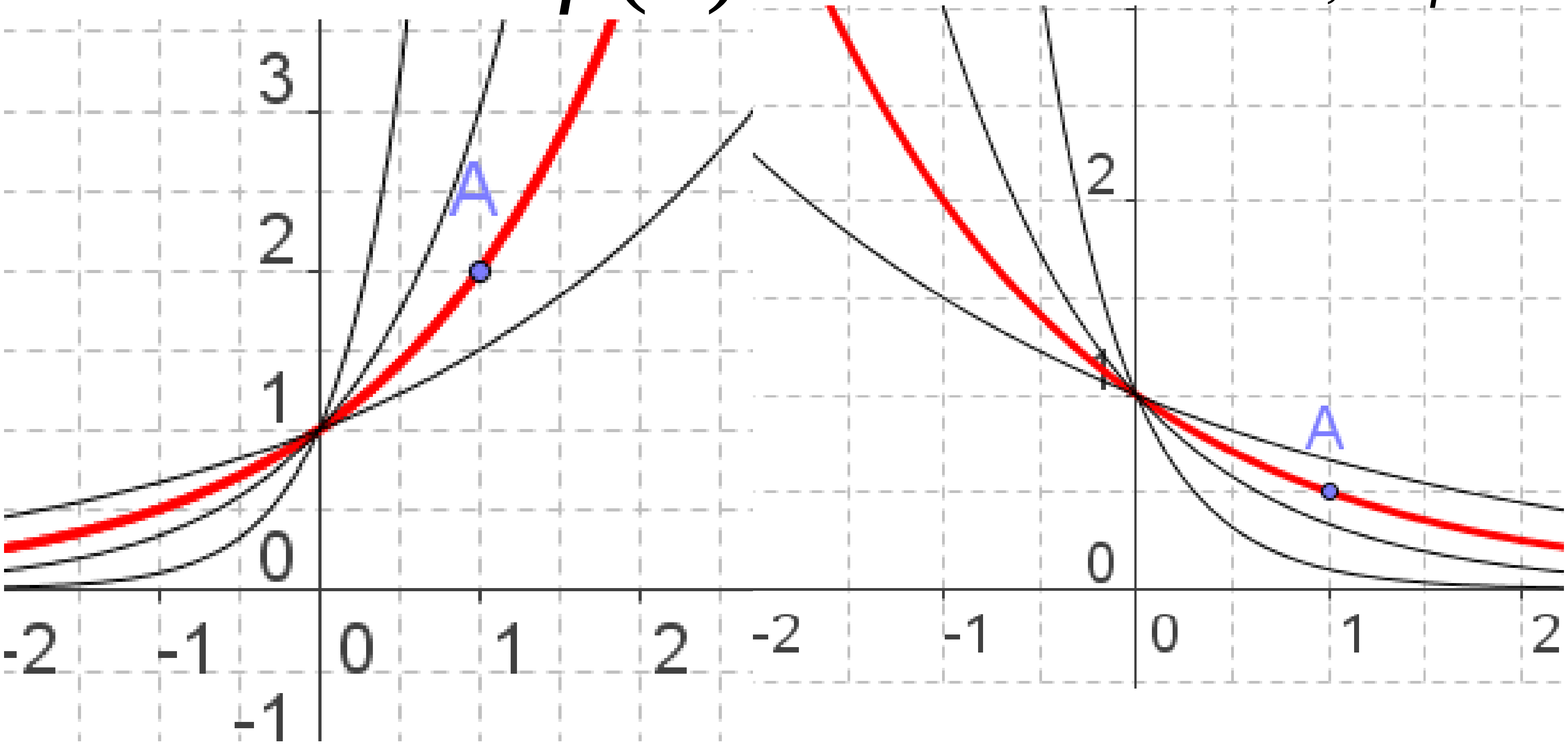

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für Basis k <0 ist f nicht definiert

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Exponential Functions

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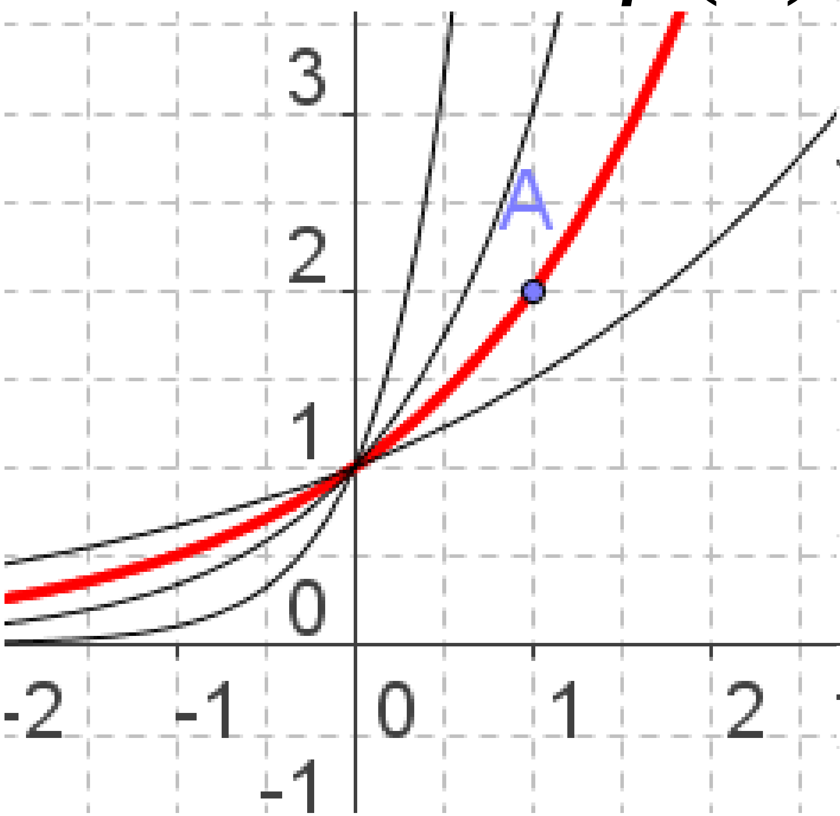
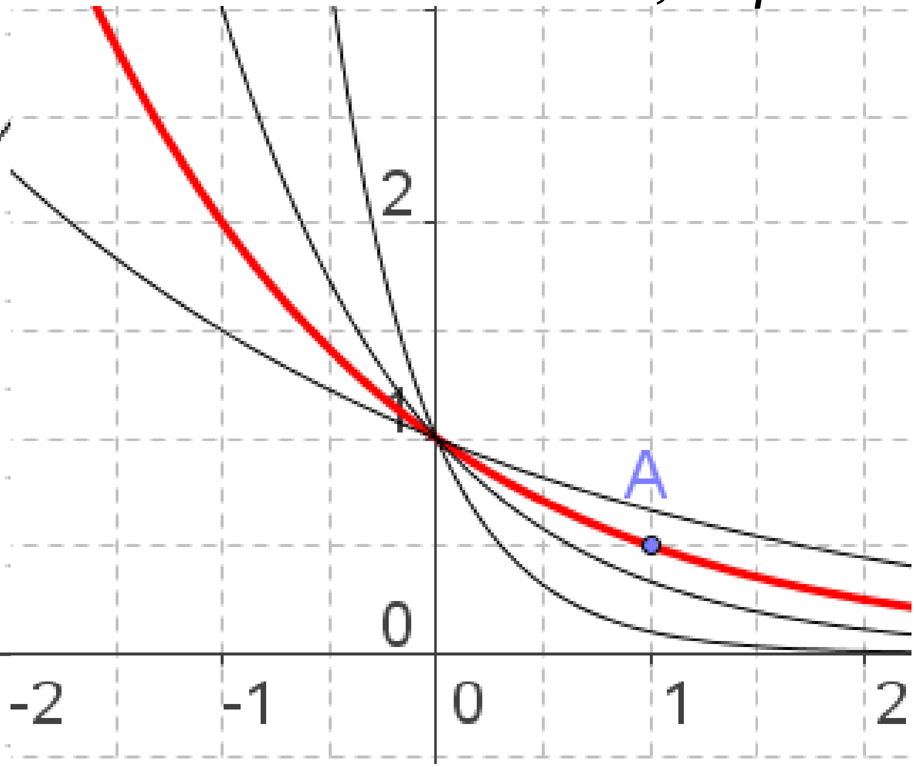


base k >1



base k with 0<k <1

for basis k <0 there is no definition for f

e-Funktion, das halbe Geheimnis

hin

$$f(x) = k^x \qquad f(x) = e^x$$

Folie 9

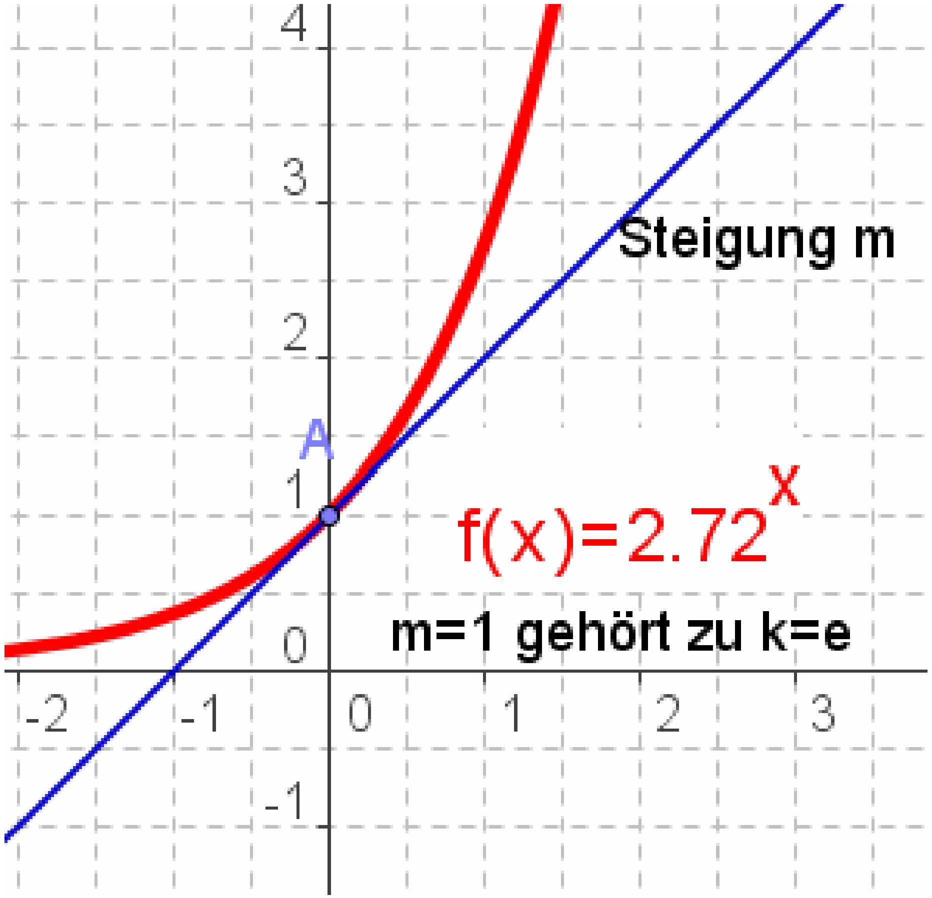
E-function, the Half Mystery

hin

$$f(x) = k^x \qquad f(x) = e^x$$

e-Funktion, das halbe Geheimnis

$$f(x) = k^x$$

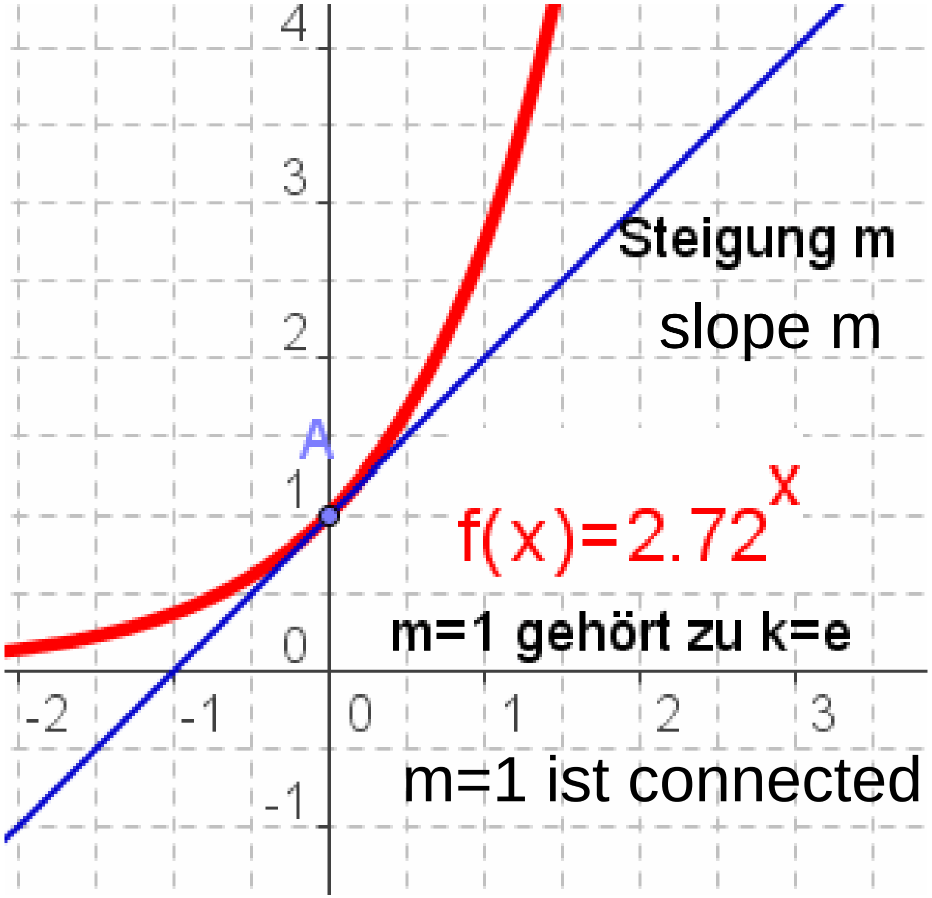


die e-Funktion ist diejenige Exponentialfunktion, die in (0/1) die Steigung 1 hat.

$$f(x) = e^x$$

E-function, the Half Mystery

$$f(x) = k^x$$



m=1 ist connected with k=e

the one and only e-function is the exponential function who has in the point (0/1) the slope 1.

$$f(x) = e^x$$

Die Welt der Umkehrfunktionen

$$y = \sqrt{x}$$

$$y = \ln(x)$$

$$y = \arcsin(x)$$

.....

$$y = \sqrt[n]{x}$$

$$y = \log_a(x)$$

The World of the Inverse Functions

$$y = \sqrt{x}$$

$$y = \ln(x)$$

$$y = \arcsin(x)$$

.....

$$y = \sqrt[n]{x}$$

$$y = \log_a(x)$$

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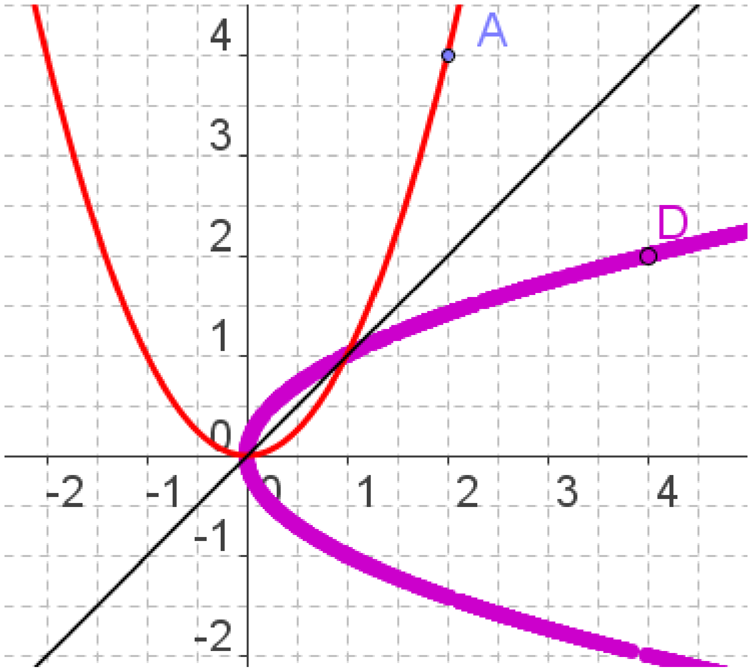
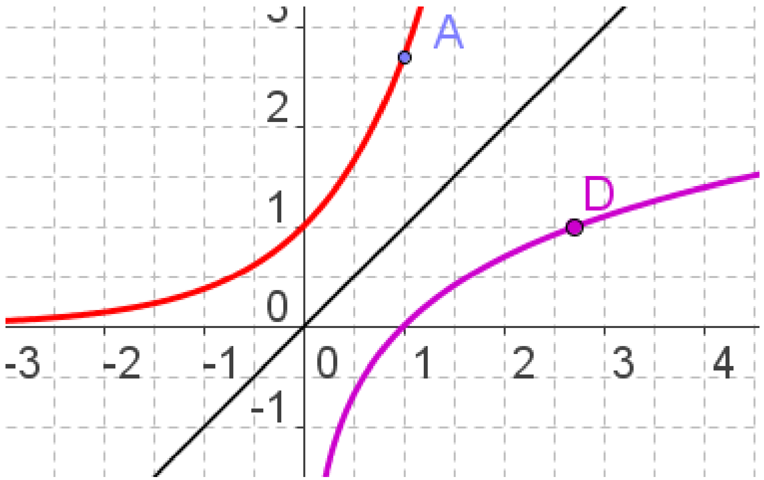
Umkehr-Fragen
Umkehr-Funktionen
Umkehr-Relationen





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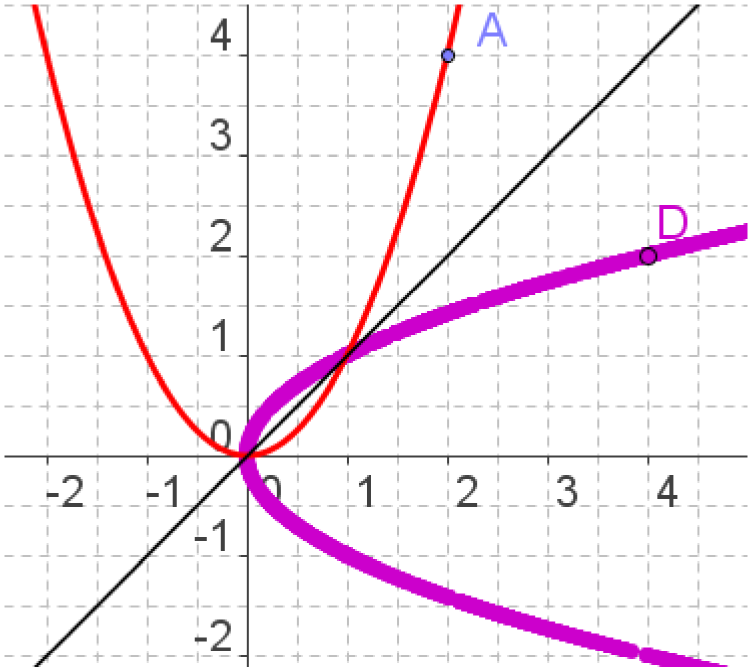
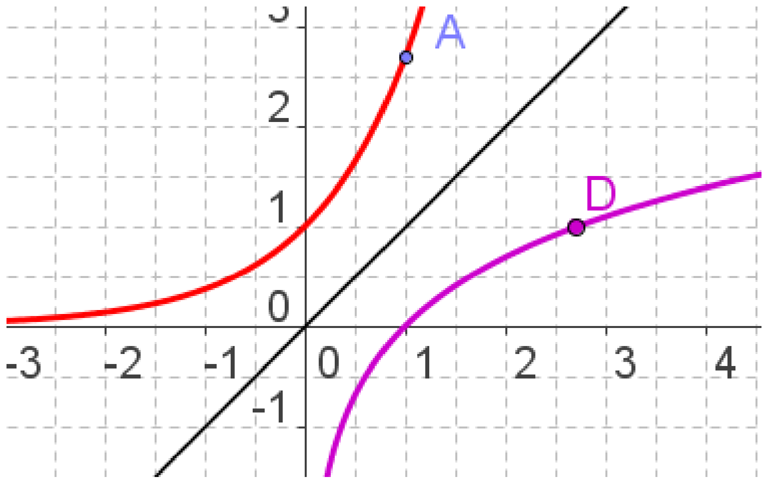
inverse questions
inverse functions
inverse relations





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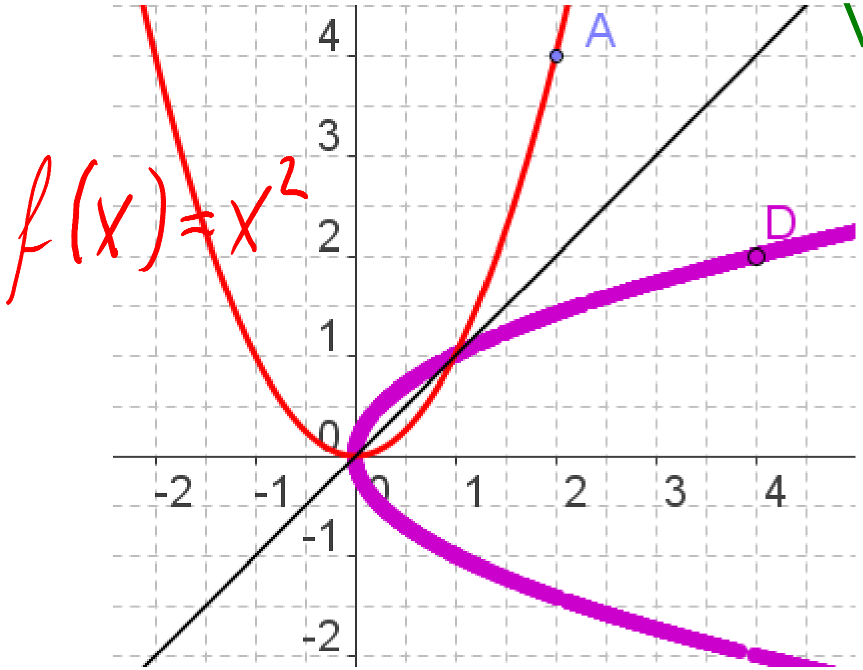
Umkehr-Fragen, Umkehr-Funktionen, Umkehr-Relationen

Frage: Welchen Wert hat f an der Stelle 2?

Antwort: 4 ist der Wert, f(2)=4

Umkehrfrage: An welchen Stellen hat f hat den Wert 4?

Antwort: +2 und -2 sind Lösungen, f(+2)=4 und f(-2)=4



Visualisierung der Umkehrfrage:

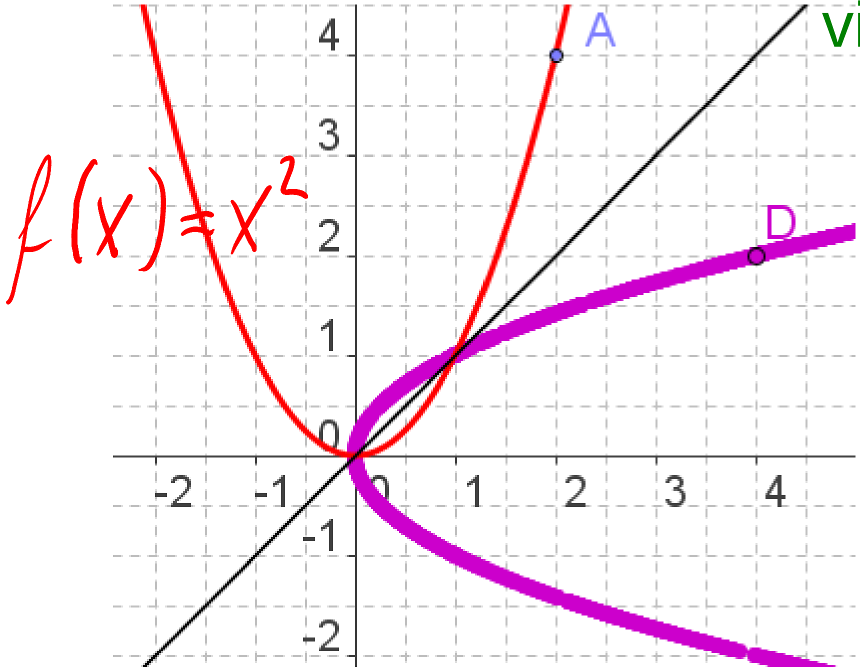
inverse questions, inverse functions, inverse relations

question: Which is the value of f at abscissa 2?

answer: 4 is the value, f(2)=4

inverse question: at which postions has f the value 2?

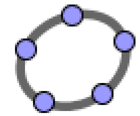
answer: +2 and -2 are the solutions, f(+2)=4 und f(-2)=4



visualisation of the inverse question:

Umkehr-Fragen, Umkehr-Funktionen, Umkehr-Relationen

Frage: Welchen Wert hat f an der Stelle 2?



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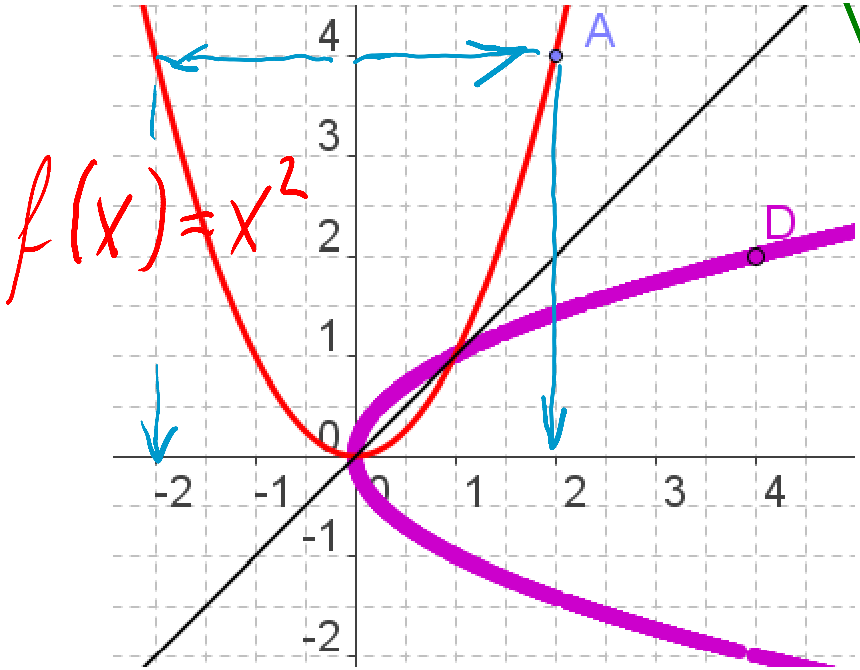

Visualisierung der Umkehrfrage:

Gehe von der y-Achse zur Kurve und dann zur x-Achse

inverse questions, inverse functions, inverse relations

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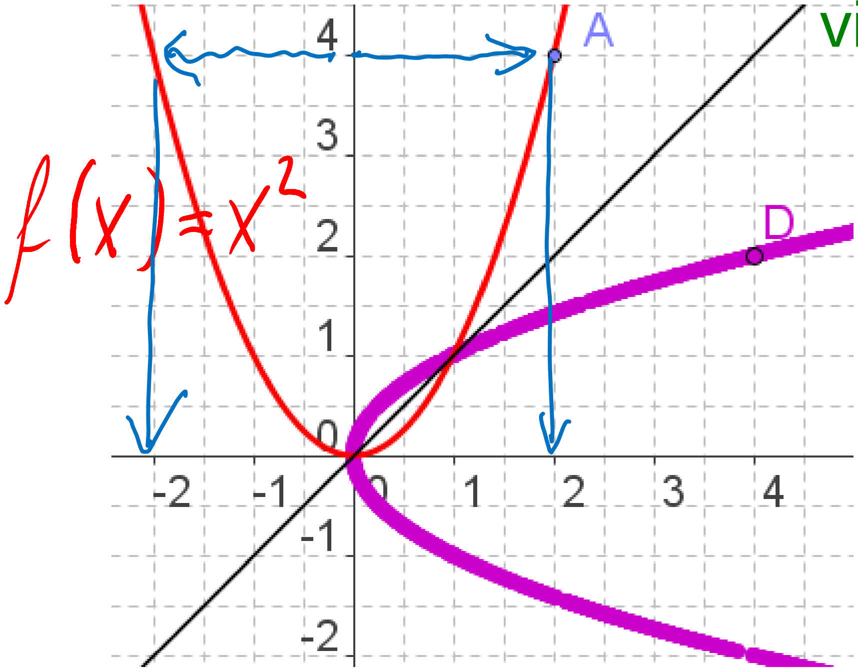

visualisation of the inverse question:

draw from the y-axis to the curve
and then draw to the x-axis

Umkehr-Fragen, Umkehr-Funktionen, Umkehr-Relationen

Umkehrfkt

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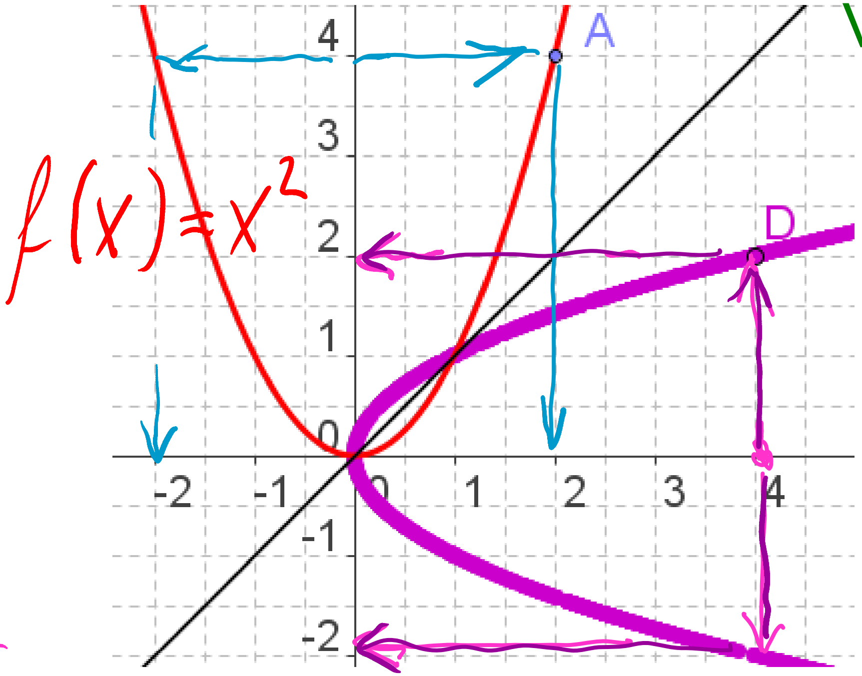


Visualisierung der Umkehrfrage:

I Gehe von der y-Achse zur Kurve und dann zur x-Achse

oder

II Gehe von der x-Achse zum Graphen der an der Winkel-halbierenden gespiegelten Kurve und dann zur y-Achse. Es ist die Umkehrrelation.

oder III $x=y^2$

III Dies ist nur eine Relation, **keine** Funktion. Der Wert ist nicht eindeutig.

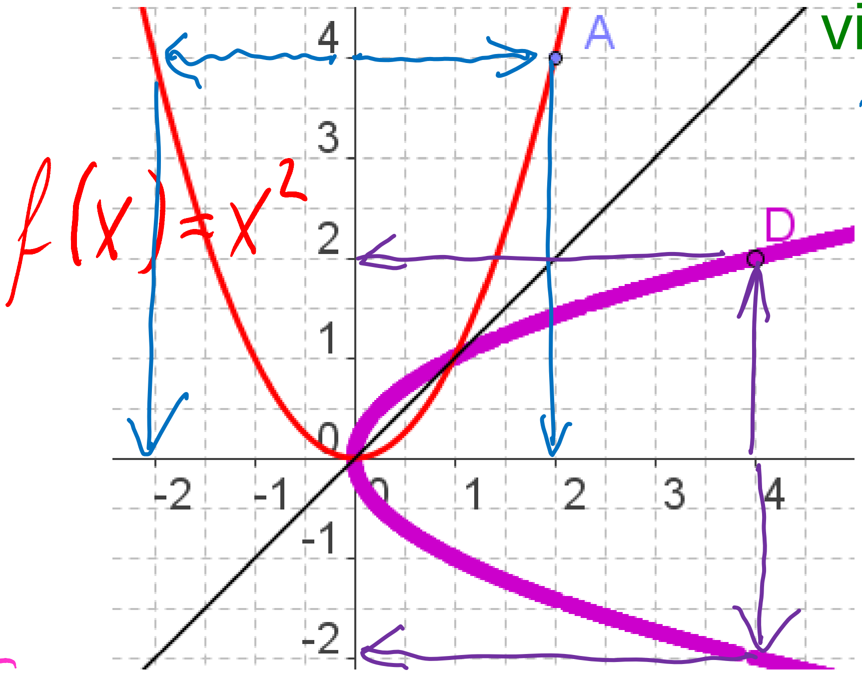
inverse questions, inverse functions, inverse relations

question: Which is the value of f at abscissa 2?

answer: 4 is the value, f(2)=4

inverse question: at which postions has f the value 2?

answer: +2 and -2 are the solutions, f(+2)=4 und f(-2)=4



visualisation of the inverse question:

I draw from the y-axis to the curve and then draw to the x-axis

or

II at first reflect the curve with the angle bisection line y=x then draw from the x-axis to this curve and than draw to the y-axis

or

III $x=y^2$

III This is only a relation, not an equation of a function, the y-value is not unique.

Umkehr-Fragen, Umkehr-Funktionen, Umkehr-Relationen

Umkehrfkt

Frage: Welchen Wert hat f an der Stelle 2?

Antwort: 4 ist der Wert, f(2)=4

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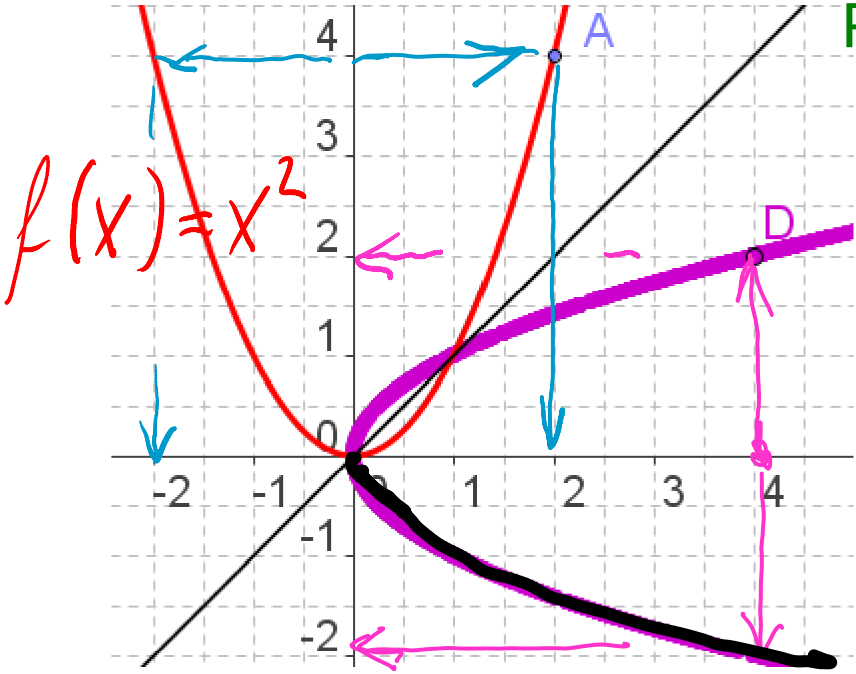


Formalisierung der Umkehrfrage:

f^{-1} = **Umkehrfunktion von** f

Bilde (hier stückweise) die Umkehrfunktion

$$g(x)=\sqrt{x}$$

$g(4)=\sqrt{4}=2$

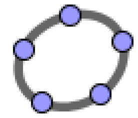
$$h(x)=-\sqrt{x}$$

$h(4)=-2$

inverse questions, inverse functions, inverse relations

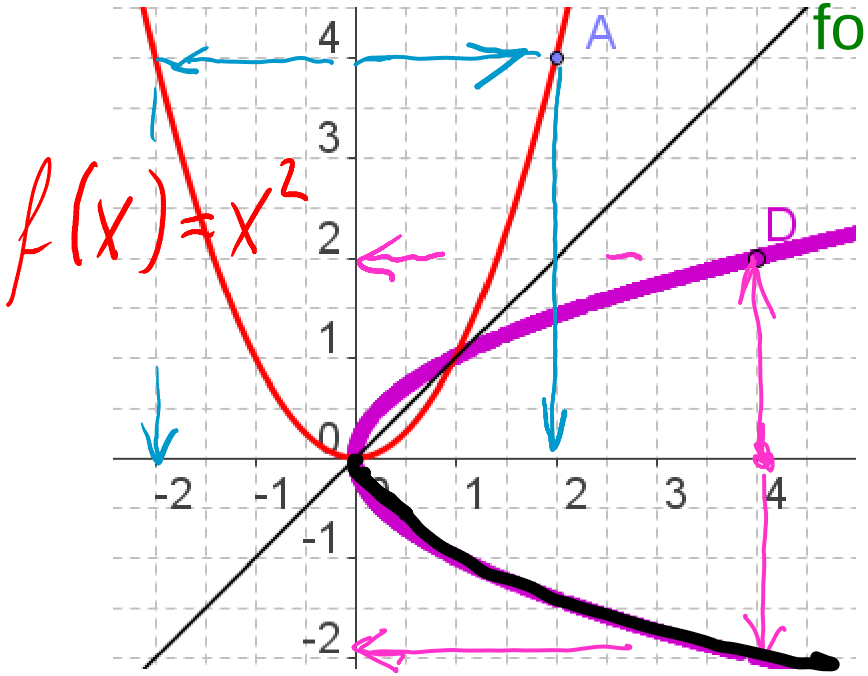
question: Which is the value of f at abscissa 2?

answer: 4 is the value, $f(2)=4$



inverse question: at which postions has f the value 2?

answer: +2 and -2 are the solutions, $f(+2)=4$ und $f(-2)=4$



formalisation of the inverse question

f^{-1} = **inverse function of** f

build the inverse function

it is here only piecewise possible

$$g(x)=\sqrt{x}$$

$g(4)=\sqrt{4}=2$

$$h(x)=-\sqrt{x}$$

$h(4)=-2$

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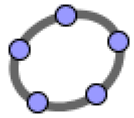
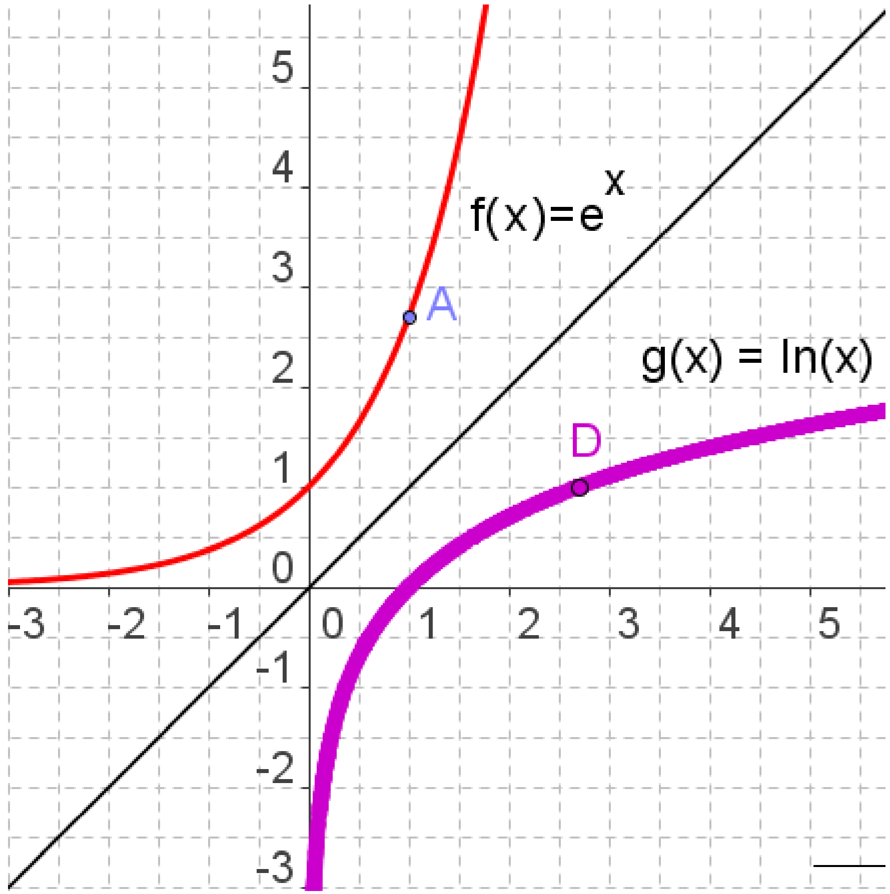
die

Exponentialfunktion

$$f(x) = e^x$$



Eulersche
e-Funktion



der natürliche
Logarithmus

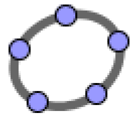
die ln-Funktion

der ln

the one and only

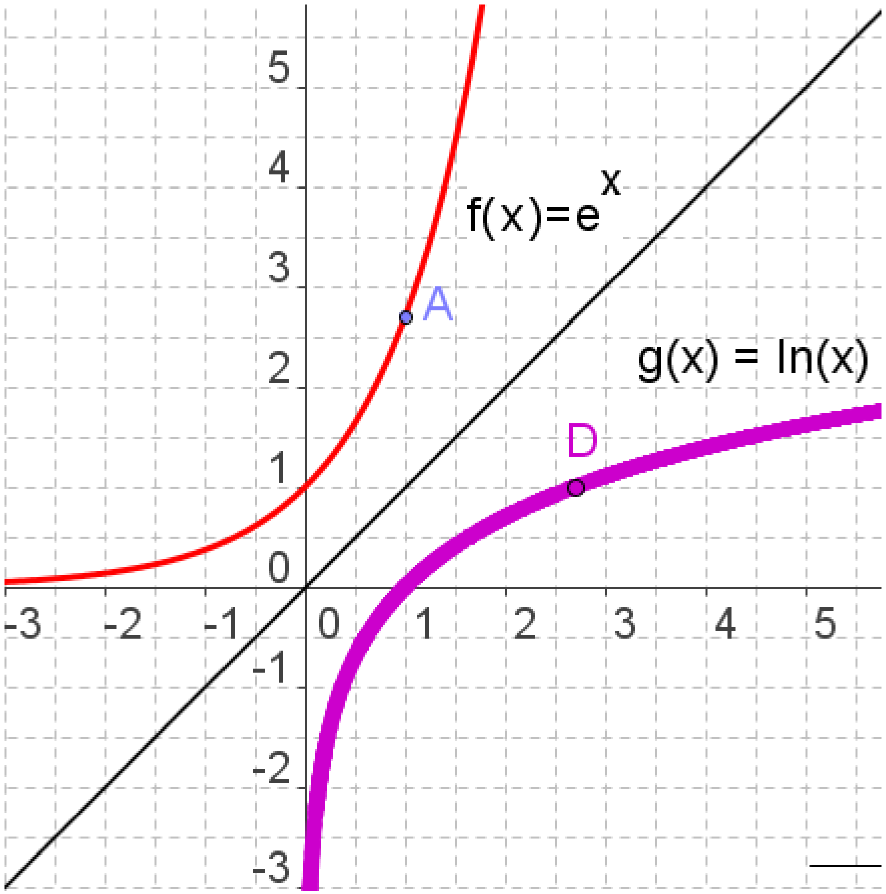
Exponential Function

$$f(x) = e^x$$



Umkehrfkt

Euler‘s
e-Funktion



the natural
logarithm

the ln-function

the ln

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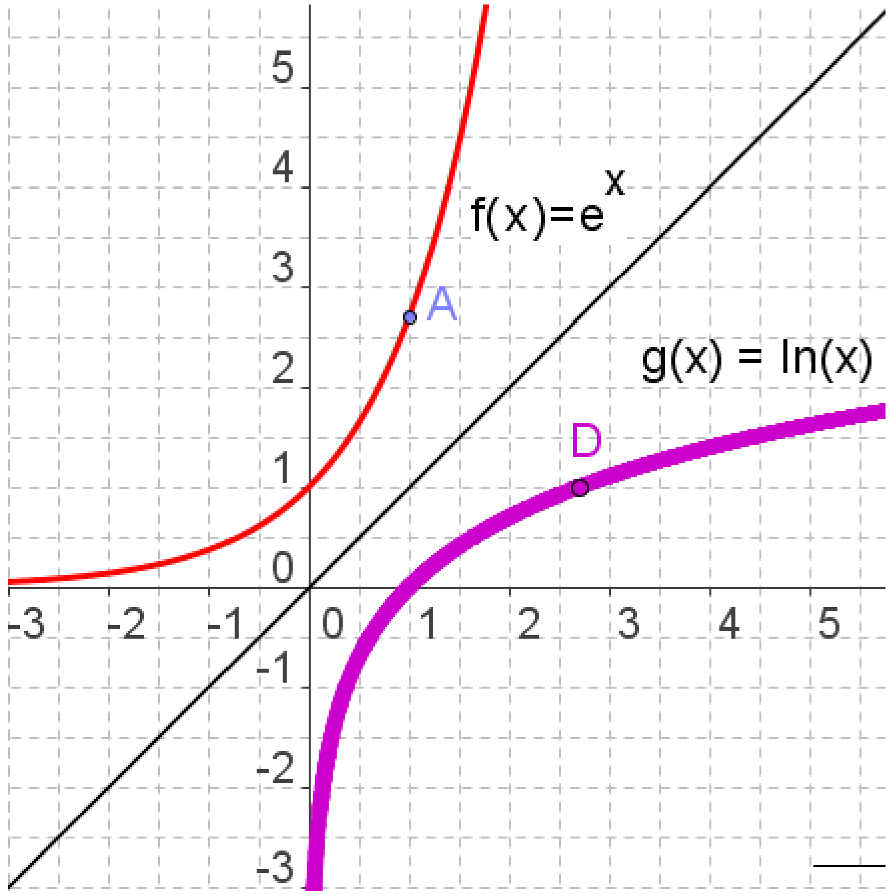
die Exponentialfunktion

$$f(x) = e^x \qquad e^{\ln(x)} = x$$

Umkehrfkt

$\ln(e^x) = x$



Eulersche e-Funktion

f

$f(1) = e^1 = e$

$f(0) = e^0 = 1$

der natürliche Logarithmus

die ln-Funktion

der ln

$f^{-1}(x) = \ln(x)$

$\ln(e) = 1$

$\ln(1) = 0$

the one and only

Exponential Function

$$f(x) = e^x$$

$e^{\ln(x)} = x$

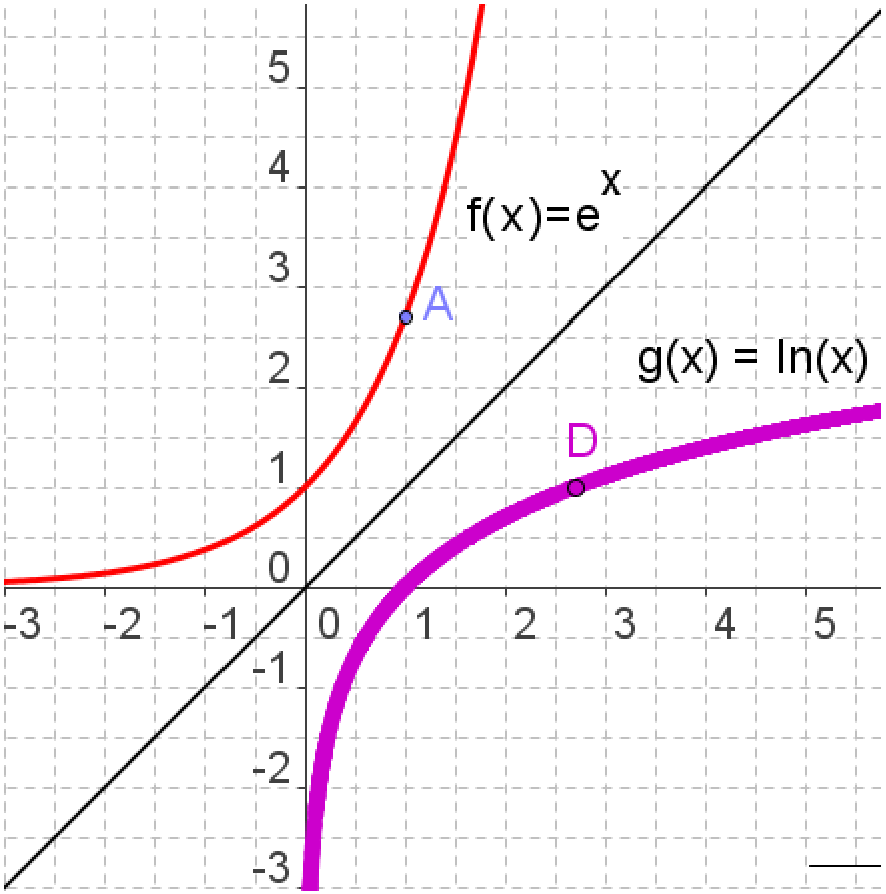
Umkehrfkt

$\ln(e^x) = x$

the natural logarithm

Euler's e-Funktion

f



the ln-function

the ln

$f^{-1}(x) = \ln(x)$

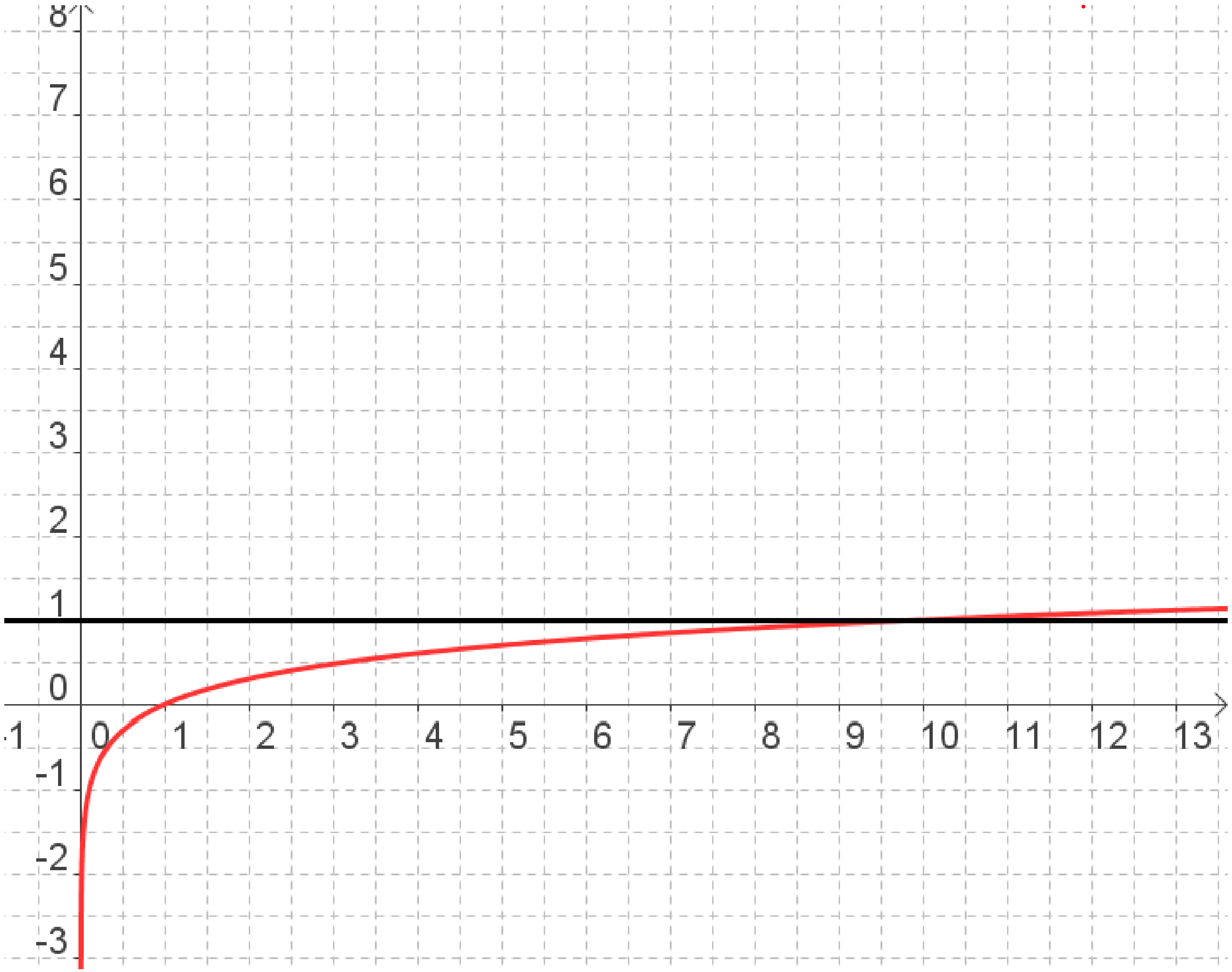
$f(1) = e^1 = e$

$f(0) = e^0 = 1$

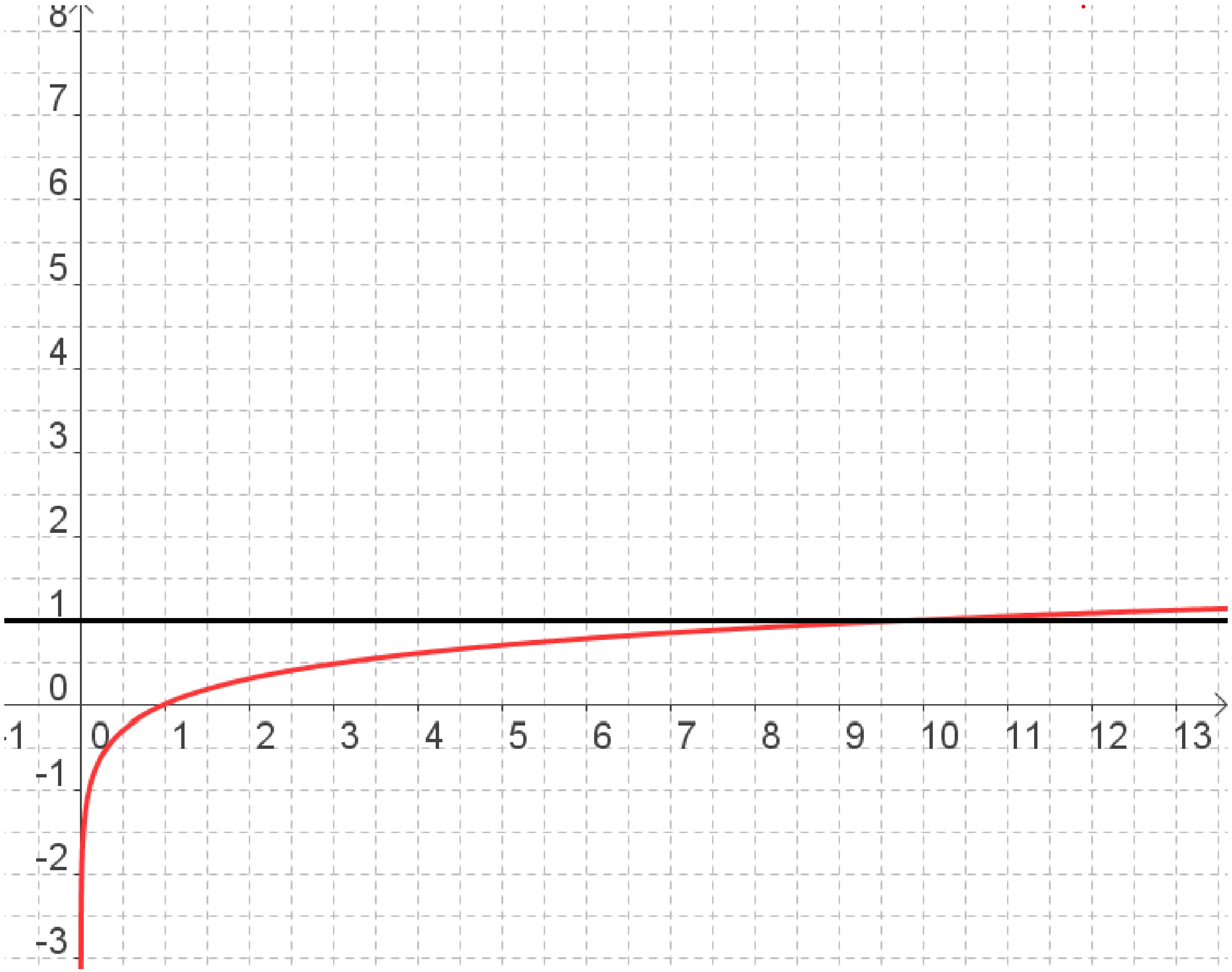
$\ln(e) = 1$

$\ln(1) = 0$

Wie langsam wächst der Logarithmus?



How Slow the Logarithm is Growing?



Umkehrfkt

Jede Funktion frisst ihre Umkehrfunktion

für x > 0

$$y = \sqrt{x}$$

$$y = \ln(x)$$

$$y = \arcsin(x)$$

$$y = \sqrt[n]{x}$$

für Hauptwerte

$$y = \log_a(x)$$

Every Function Feeds her Inverse Function

für $x > 0$

$$y = \sqrt{x}$$

$$y = \ln(x)$$

$$y = \arcsin(x)$$

$$y = \sqrt[n]{x}$$

für Hauptwerte

$$y = \log_a(x)$$

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Jeder Funktion frisst ihre Umkehrfunktion

für $x > 0$

$$y = \sqrt{x}$$

$\sqrt{x^2} = |x|$

$(\sqrt{x})^2 = x$

$$y = \arcsin(x)$$

$\sin(\arcsin(x)) = x$

$\arcsin(\sin(x)) = x$

für Hauptwerte

$$y = \ln(x)$$

$\ln(e^x) = x$

$e^{\ln x} = x$

$$y = \sqrt[n]{x}$$

$\sqrt[n]{x^n} = |x|$

$$y = \log_a(x)$$

Every Function Feeds her Inverse Function

$$y = \sqrt{x}$$

$$\sqrt{x^2} = |x|$$

$$(\sqrt{x})^2 = x$$

$$y = \arcsin(x)$$

$$\sin(\arcsin(x)) = x$$

$$\arcsin(\sin(x)) = x$$

für main values

für $x > 0$

$$y = \ln(x)$$

$$\ln(e^x) = x$$

$$e^{\ln x} = x$$

$$y = \sqrt[n]{x}$$

$$\sqrt[n]{x^n} = |x|$$

$$y = \log_a(x)$$

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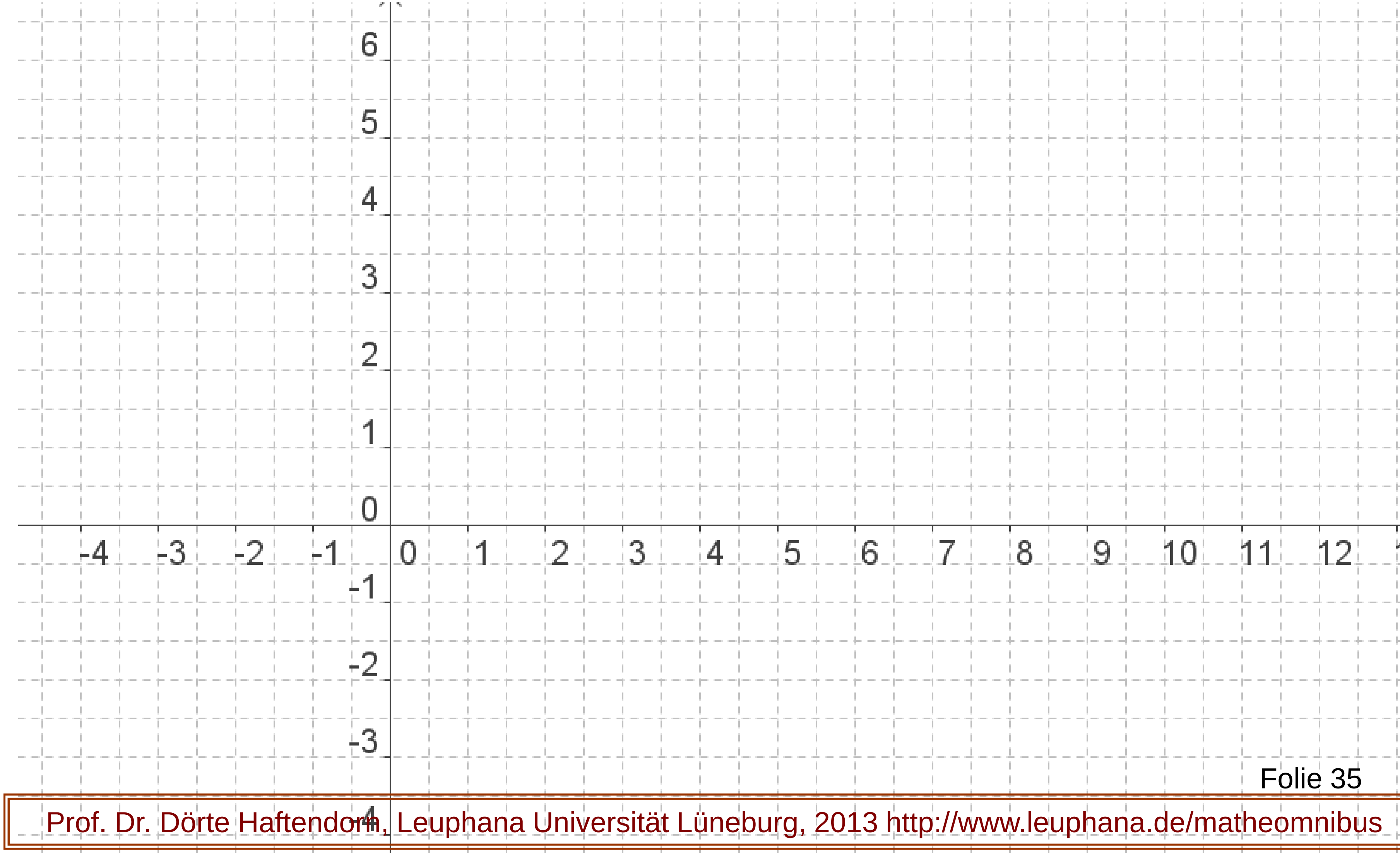
leer

Übung mit Funktionsgraphen

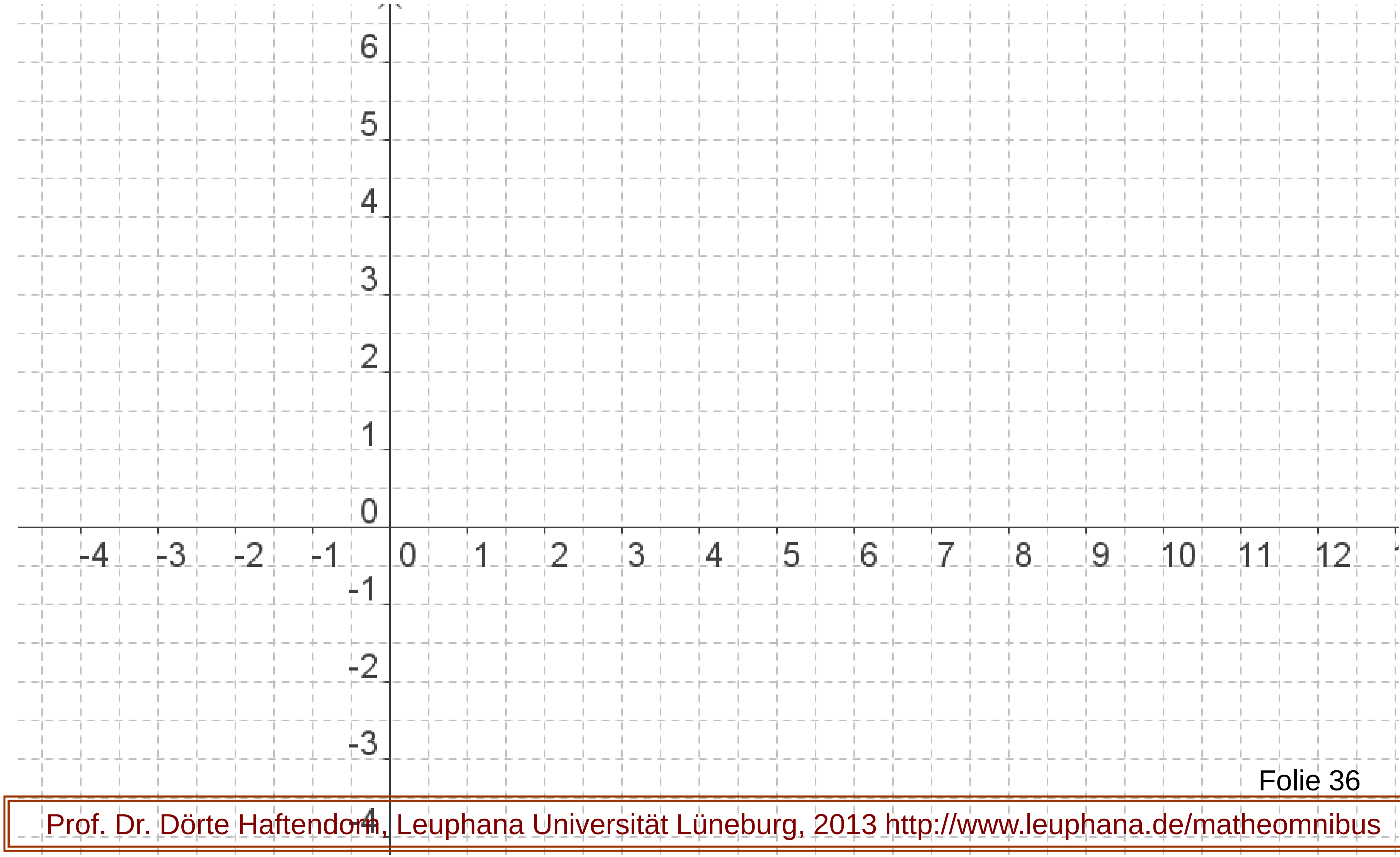
$y = e^{x}$ $y = e^{-x}$ $y = e^{x-2}$ $y = -e^{x-3} - 1$ $y = \ln(x-6)$



leer

Practise with Graphs of Functions

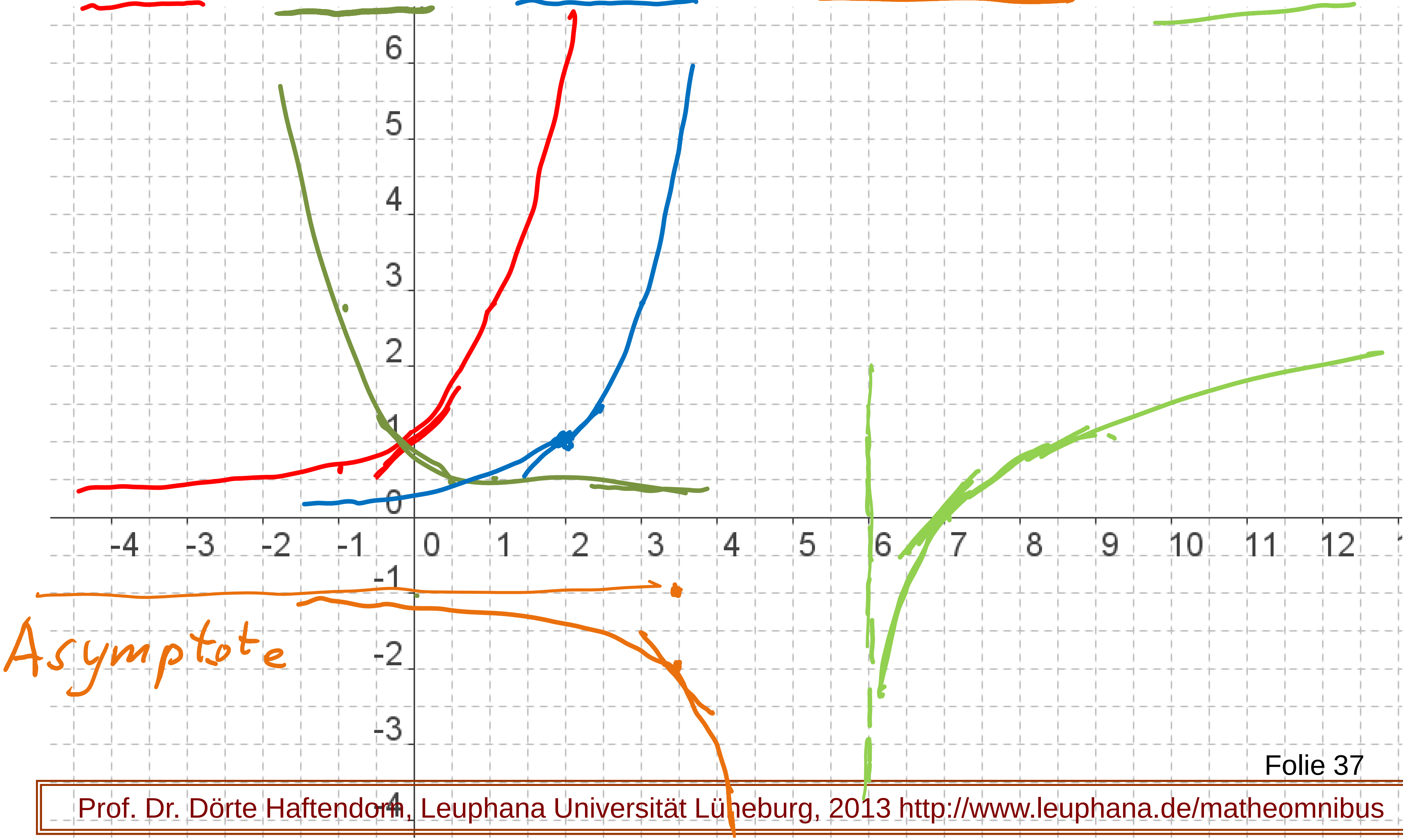
$y = e^{x}$ $\quad y = e^{-x}$ $\quad y = e^{x-2}$ $\quad y = -e^{x-3} - 1$ $\quad y = \ln(x-6)$



Übung mit Funktionsgraphen

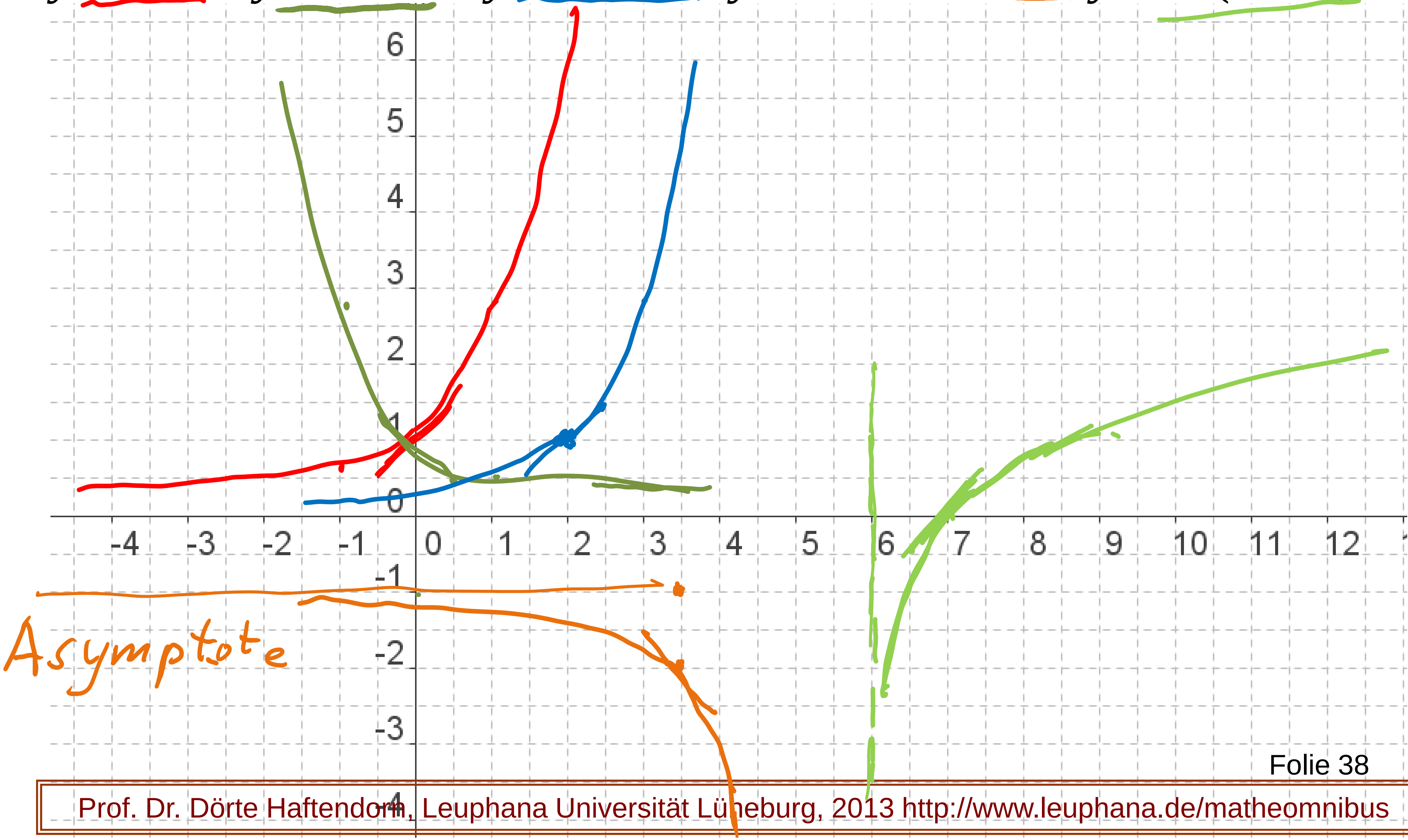
leer

$y = e^x$ $y = e^{-x}$ $y = e^{x-2}$ $y = -e^{x-3} - 1$ $y = \ln(x-6)$



practise with graphs of functions

$y = e^x$ $\quad y = e^{-x}$ $\quad y = e^{x-2}$ $\quad y = -e^{x-3} - 1$ $\quad y = \ln(x-6)$



Funktionsgleichung y = f(x)

Grundtypen

GeoGebra

Potenzfunktion

$f(x) = x^k$ $\quad f^{-1} = g$

Wurzelfunktion

$g(x) = \sqrt[k]{x}$

Exponentialfunktion

$f(x) = e^x$ $\quad f^{-1} = g$

Logarithmus

$g(x) = \ln(x)$

Trigonometrische Funktion

$f(x) = \sin(x)$ $\quad f^{-1} = g$

Arcus-Funktion

$g(x) = \arcsin(x)$

$= \text{INV} \sin(x)$

Equation of a Function $y = f(x)$

main types

GeoGebra

power function

$f(x) = x^k$ $\quad f^{-1} = g$

root function

$g(x) = \sqrt[k]{x}$

exponential function

$f(x) = e^x$ $\quad f^{-1} = g$

logarithm

$g(x) = \ln(x)$

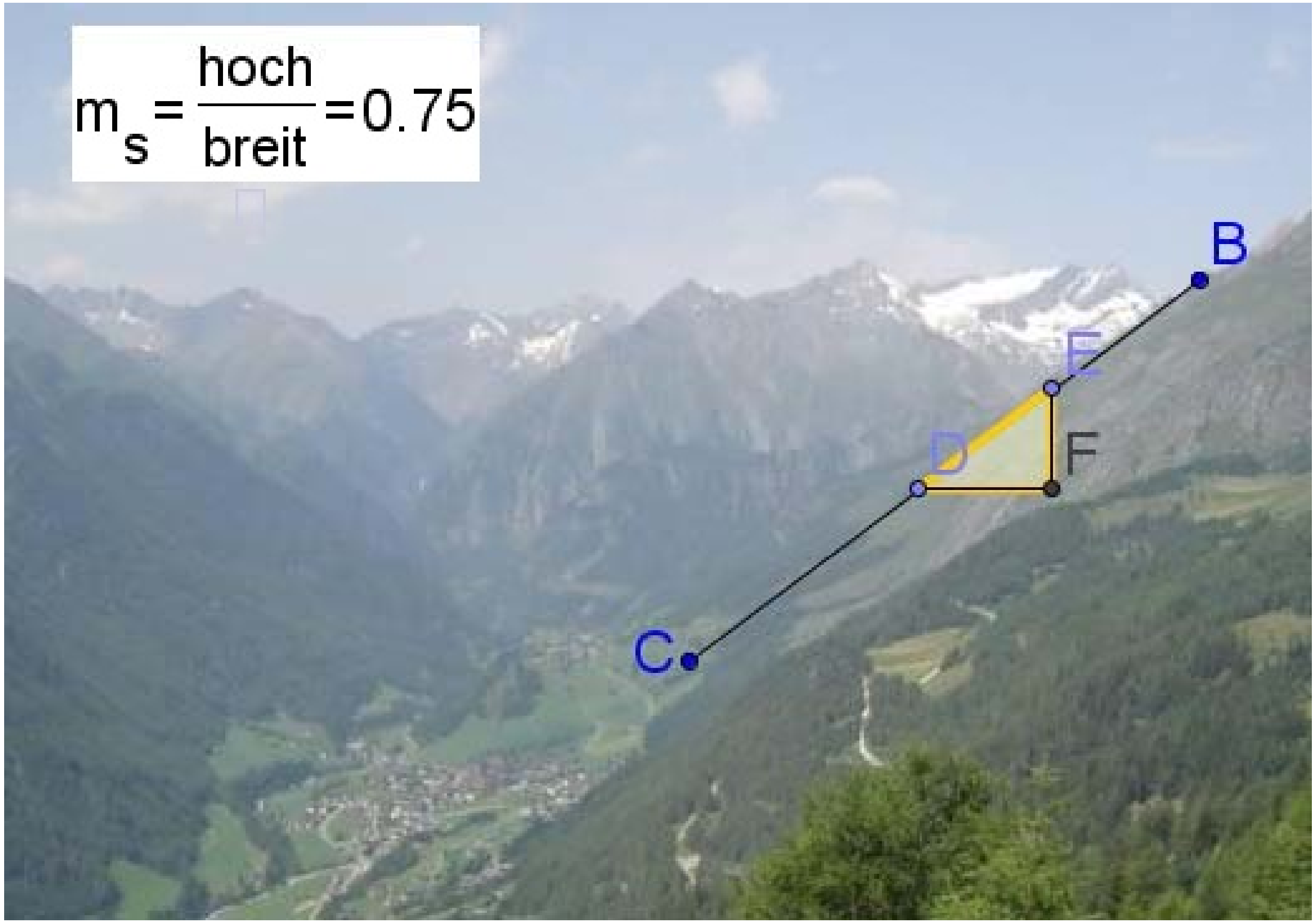
trigonometric function

$f(x) = \sin(x)$ $\quad f^{-1} = g$

arc function

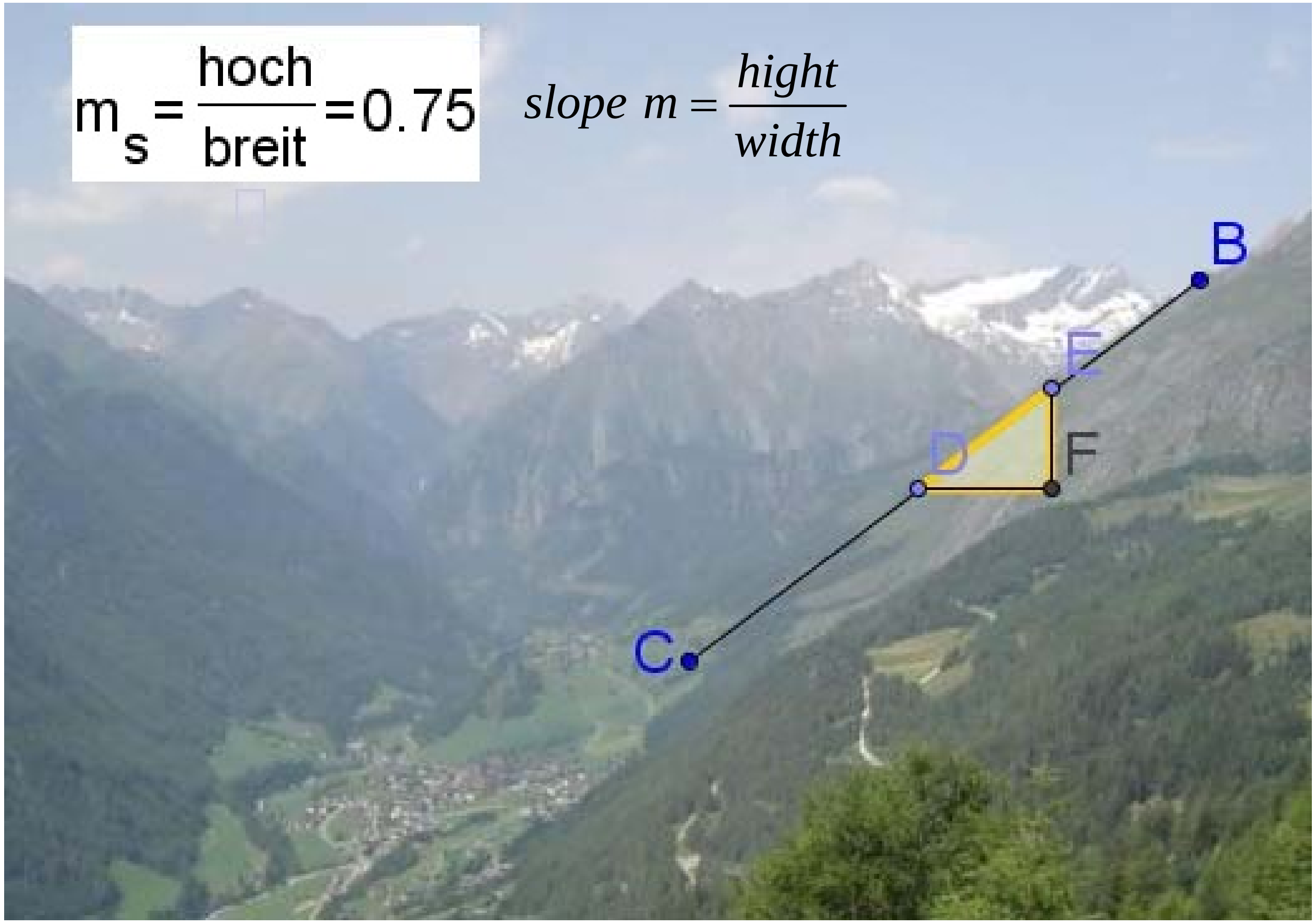
$g(x) = \mathrm{arc}\sin(x) = \mathrm{INV}\sin(x)$

Differentiale



Folie 43

Differentials

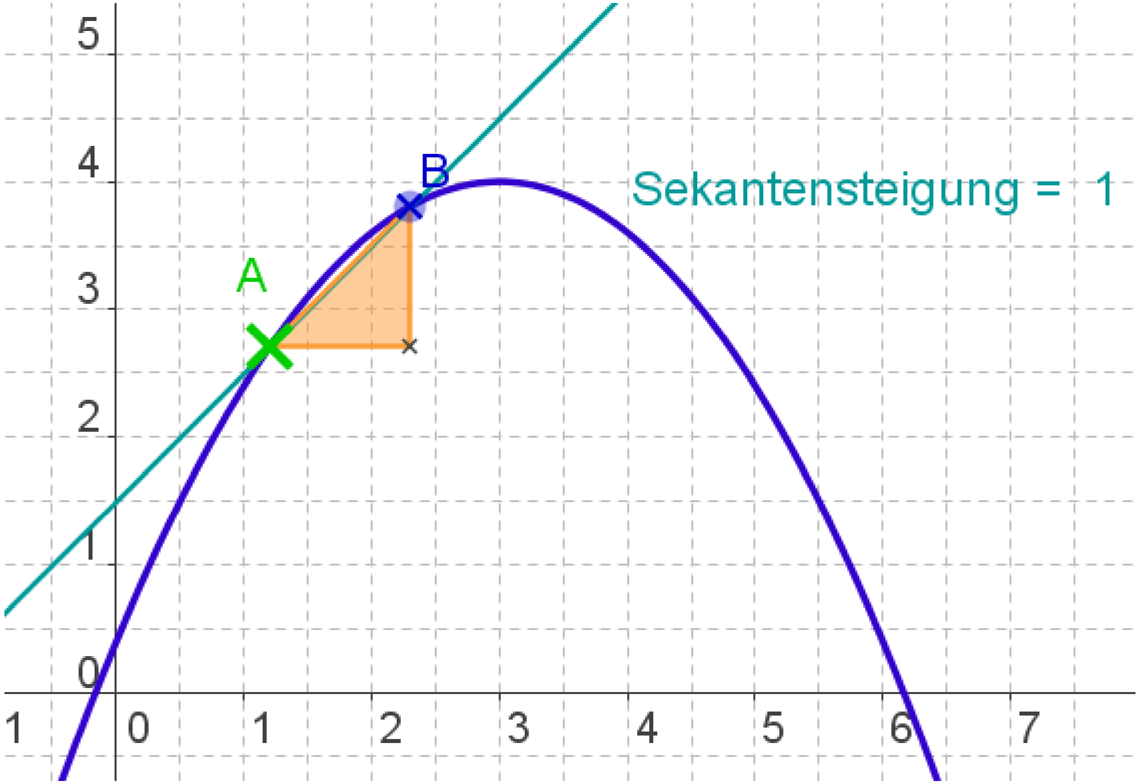


Parabel

Differentiale

Sekanten

Nur zur Vertiefung



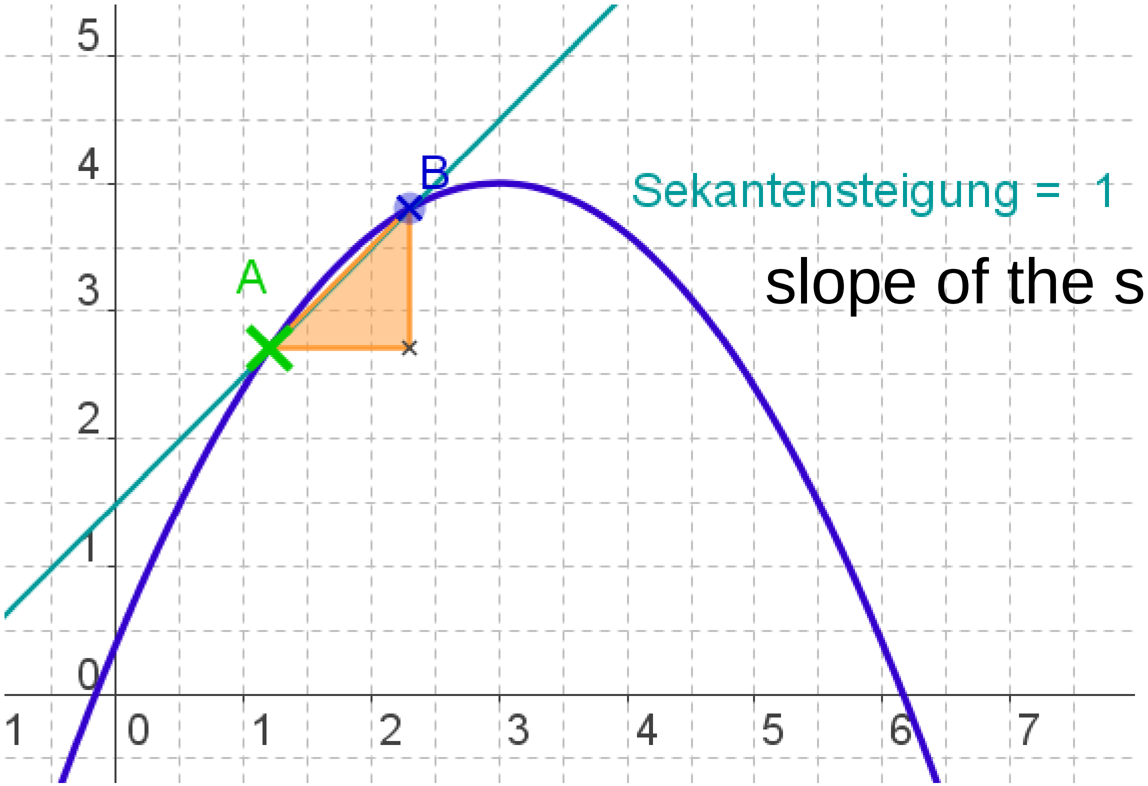
Folie 45

Parabola

Differentials

secants

only for deepening

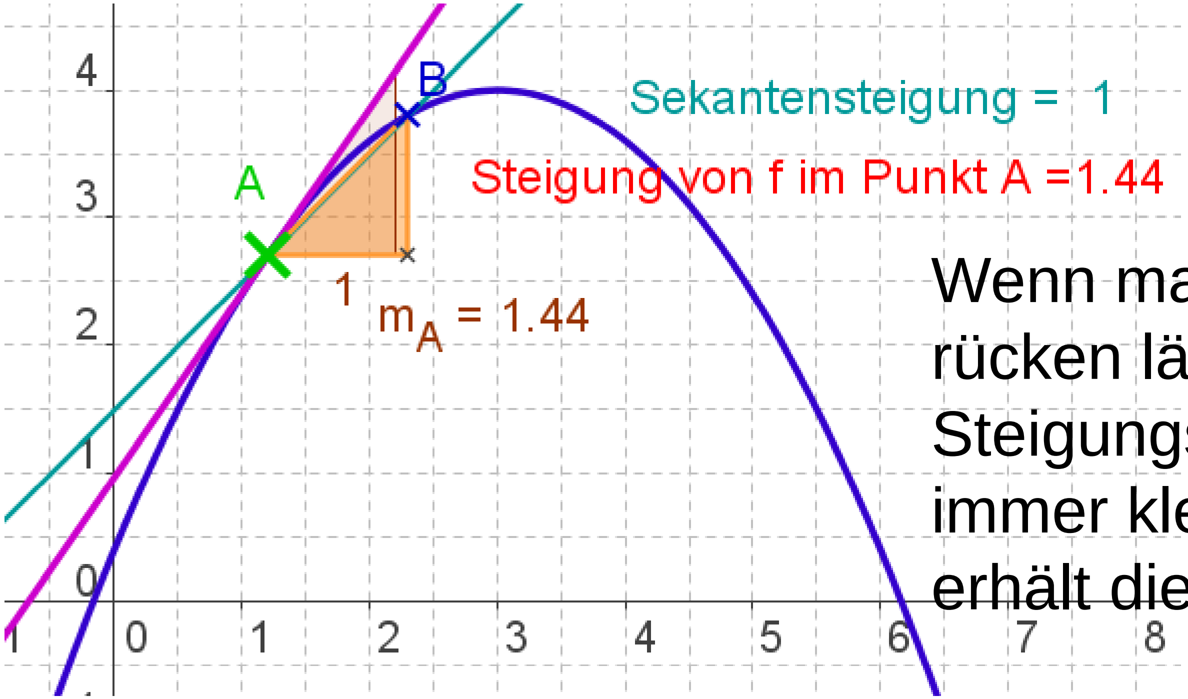


slope of the secant line

Das Differential

Also untersuchen wir für jeden Punkt einer Funktion:

welche Steigung hat die Funktion in dem Punkt?

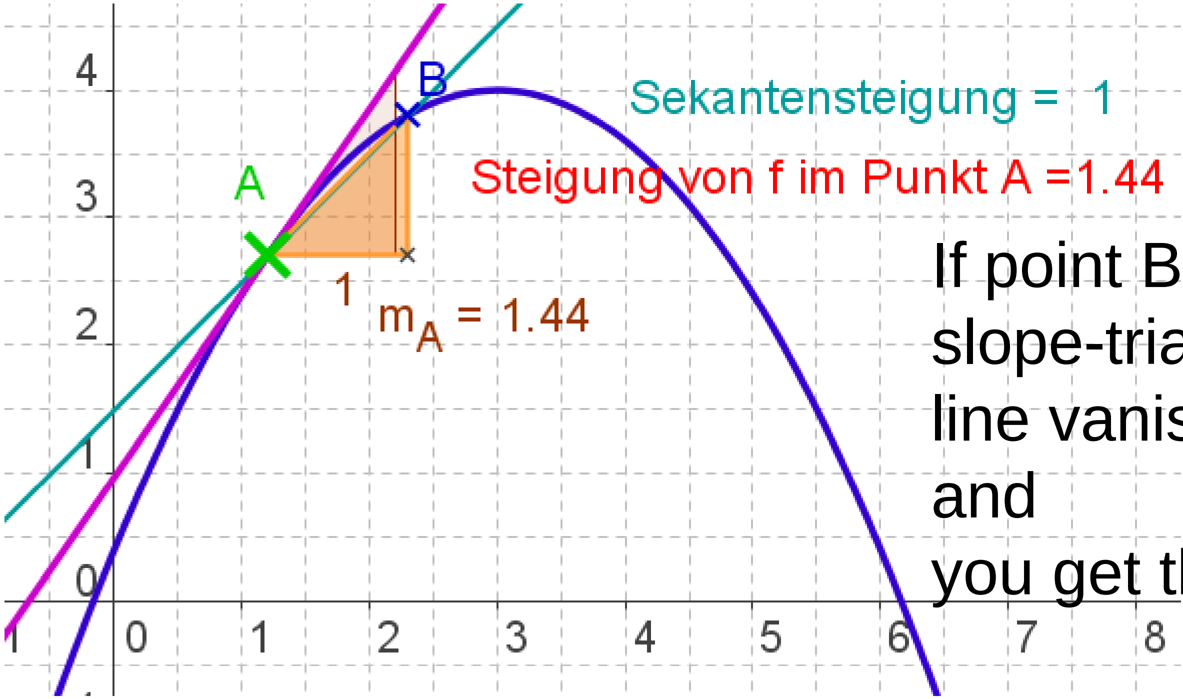


Wenn man B an A heranrücken lässt, wird das Steigungsdreieck der Sekante immer kleiner und man erhält die Tangente in A.

$$m_A = \lim_{x \to a} m_{sekante}$$

The Differential

At the end we search for every point on a function:
which slope has the function in that point?



slope of f in point A

If point B goes to point A, the slope-triangle of the secant line vanish more and more and
you get the tangent line in A.

$$m_A = \lim_{x \to a} m_{secant}$$

Das Differential

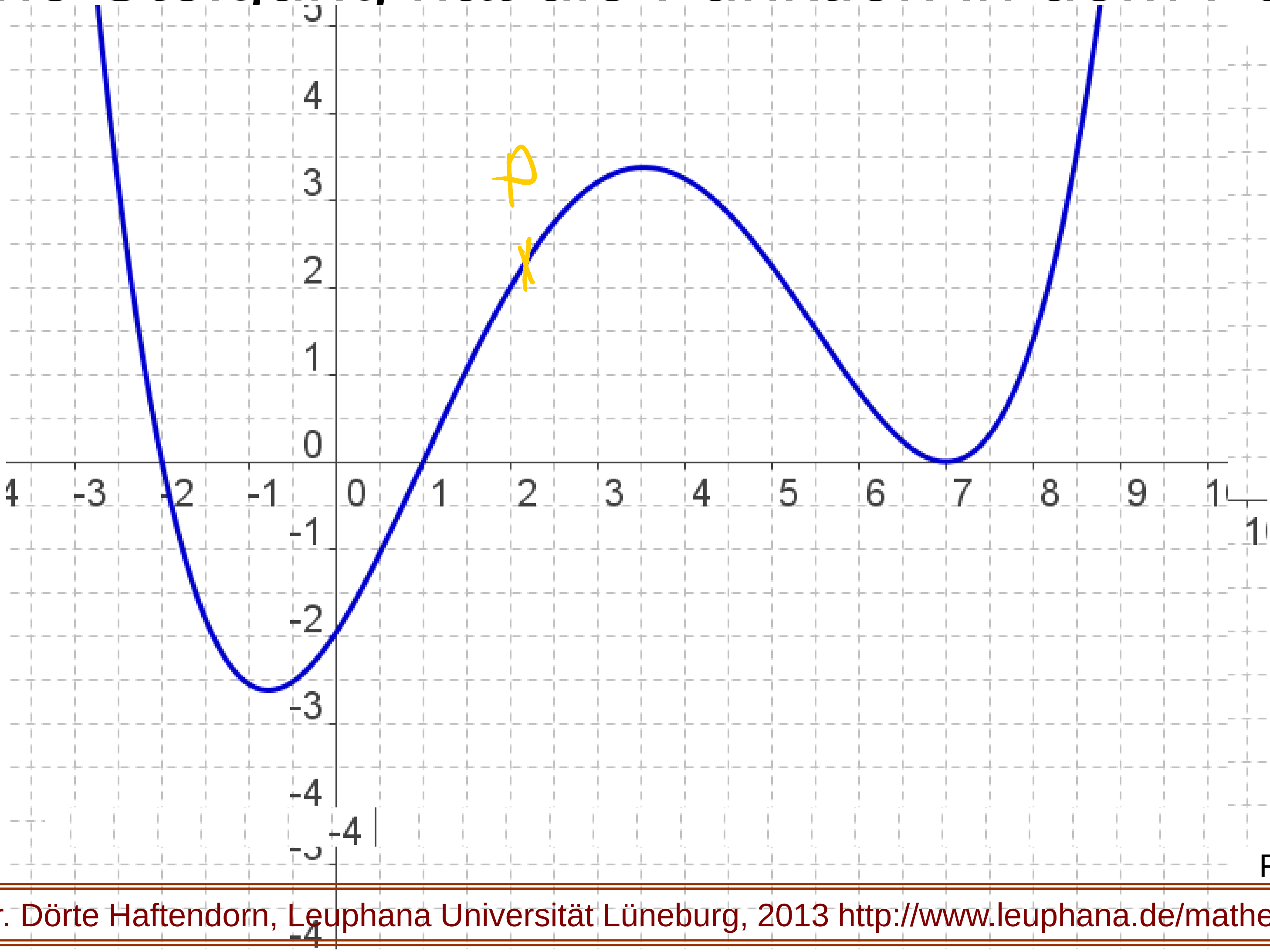
Also untersuchen wir für jeden Punkt einer Funktion:

welche Steigung hat die Funktion in dem Punkt?

Fahrrad, Bspl 2

Fahrrad pur

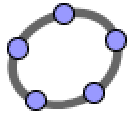
Fahrrad hier



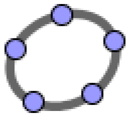
Prof. Dr. Dörte Haftendorn, Leuphana Universität Lüneburg, 2013 http://www.leuphana.de/matheomnibus

The Differential

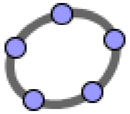
At the end we seach for every point on a function:
which slope has the function in that point?



Fahrrad, Bspl 2



Fahrrad pur



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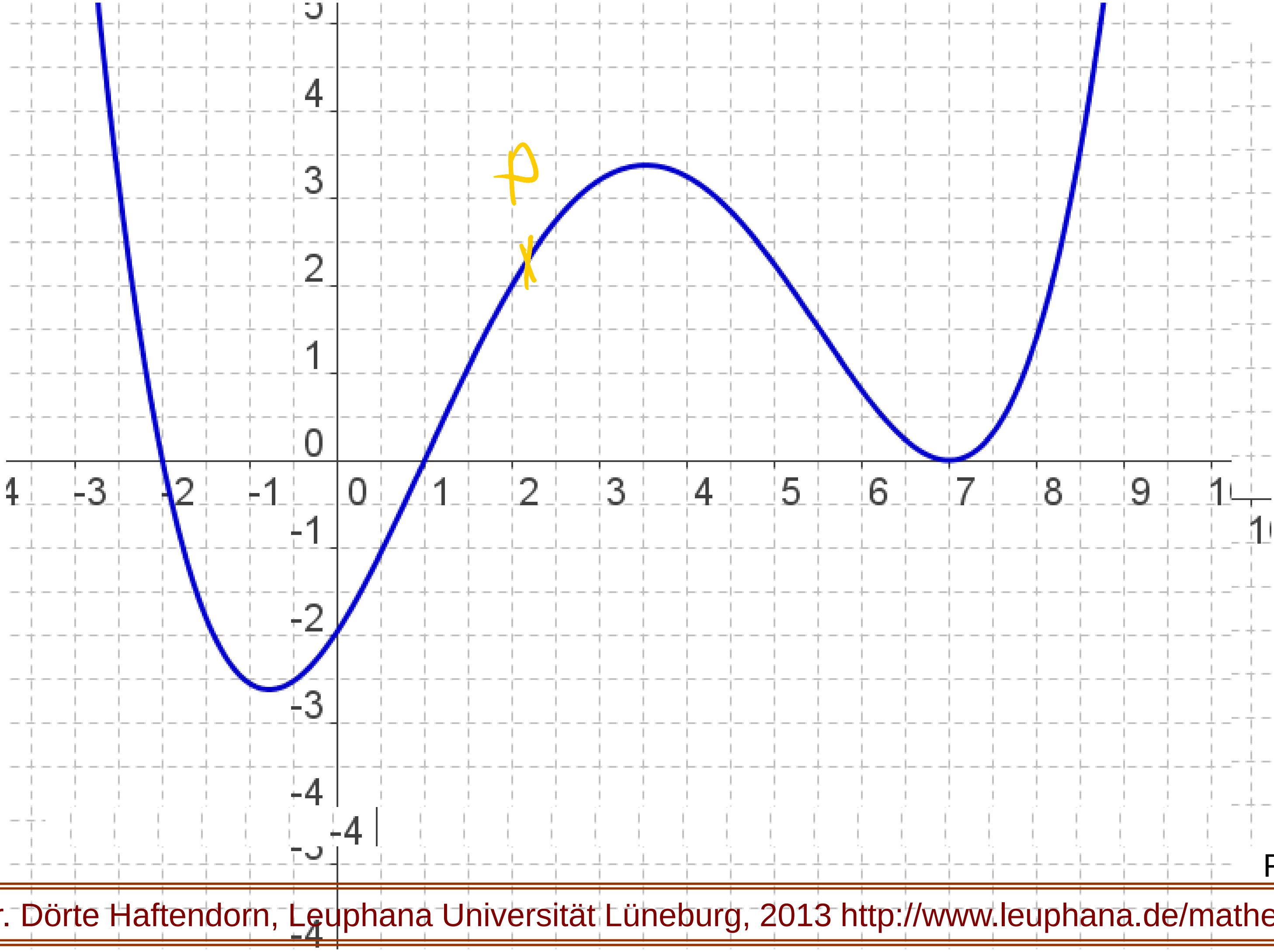


Folie 50

Das Differential

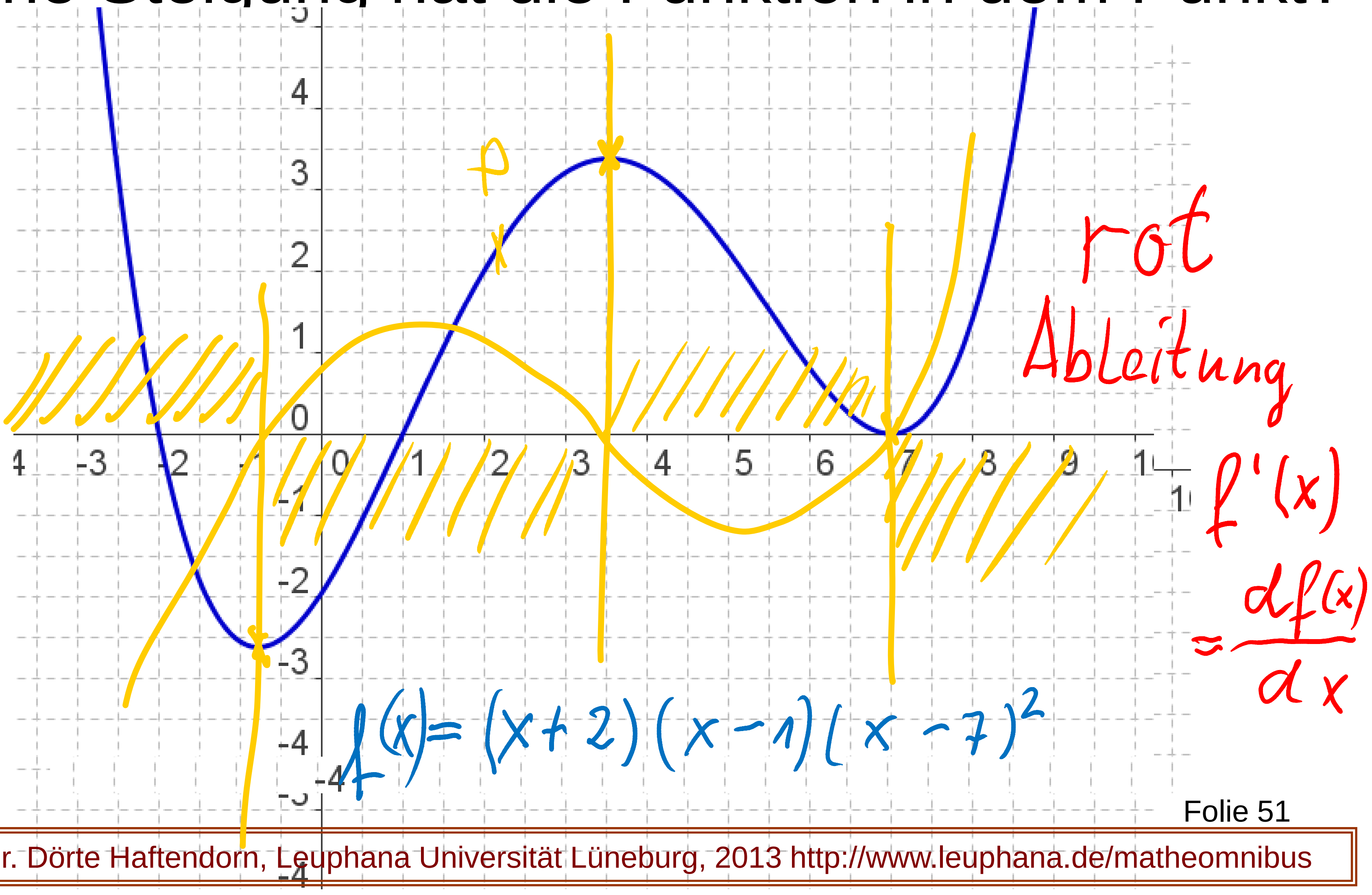
Also untersuchen wir für jeden Punkt einer Funktion:

Fahrrad, Bspl 2

welche Steigung hat die Funktion in dem Punkt?

Fahrrad pur

Fahrrad hier



Prof. Dr. Dörte Haftendorn, Leuphana Universität Lüneburg, 2013 http://www.leuphana.de/matheomnibus

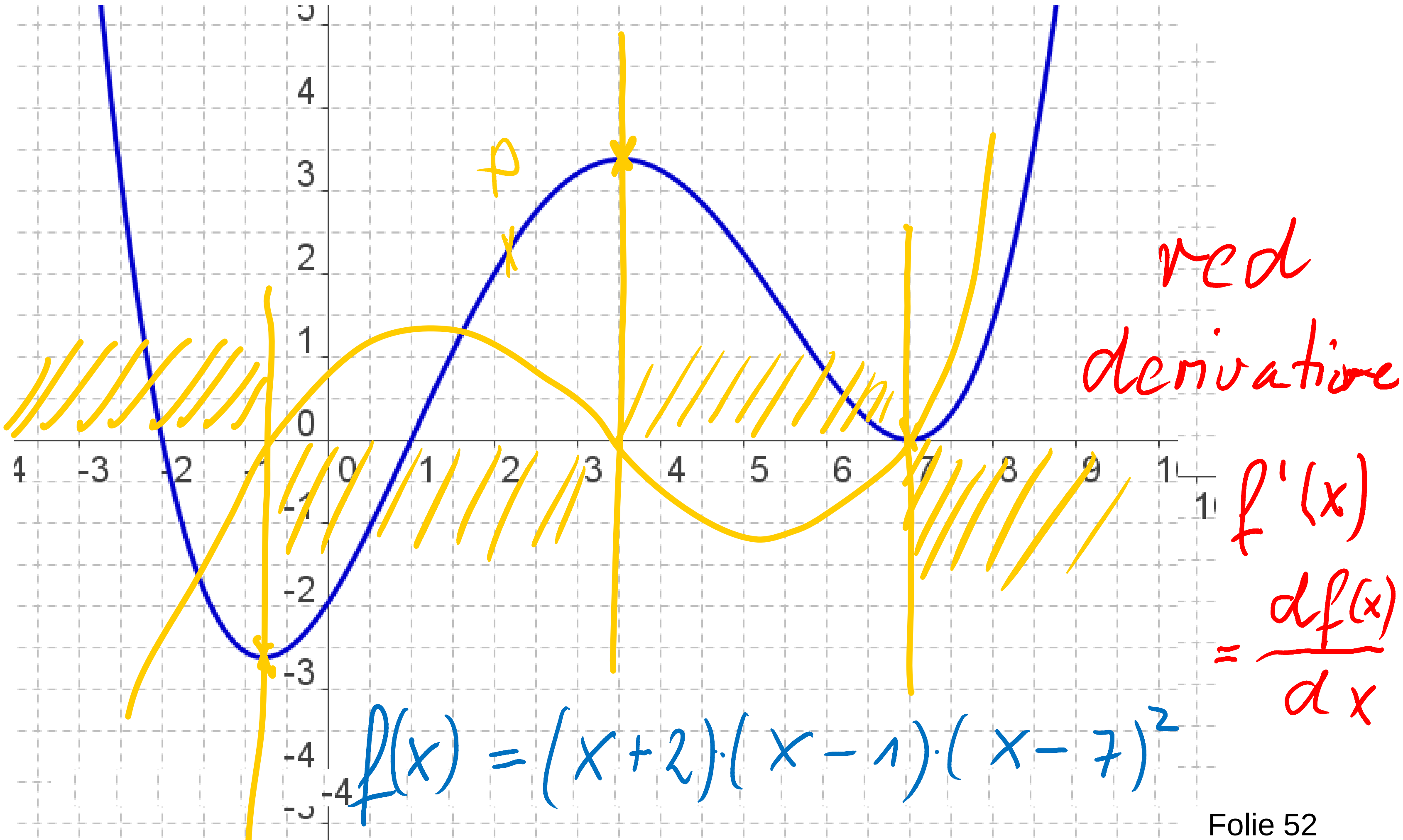
The Differential

At the end we seach for every point on a function:
which slope has the function in that point?

Fahrrad, Bspl 2

Fahrrad pur

Fahrrad hier



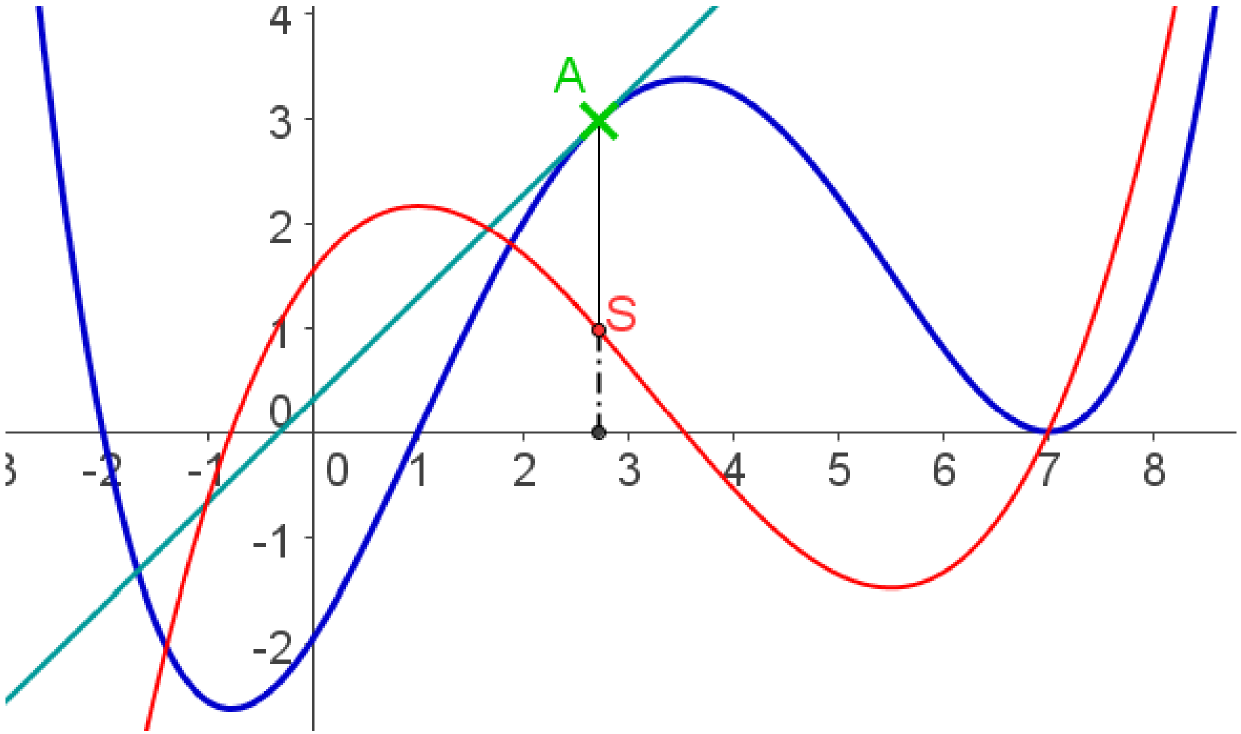
diff

Die Ableitung f ‘ ist die Funktion, die für jedes x die Steigung der Funktion f angibt.

Fahrrad, Bspl 2

Fahrrad pur

Fahrrad hier



Die rote Funktion ist also die Ableitung von der blauen.

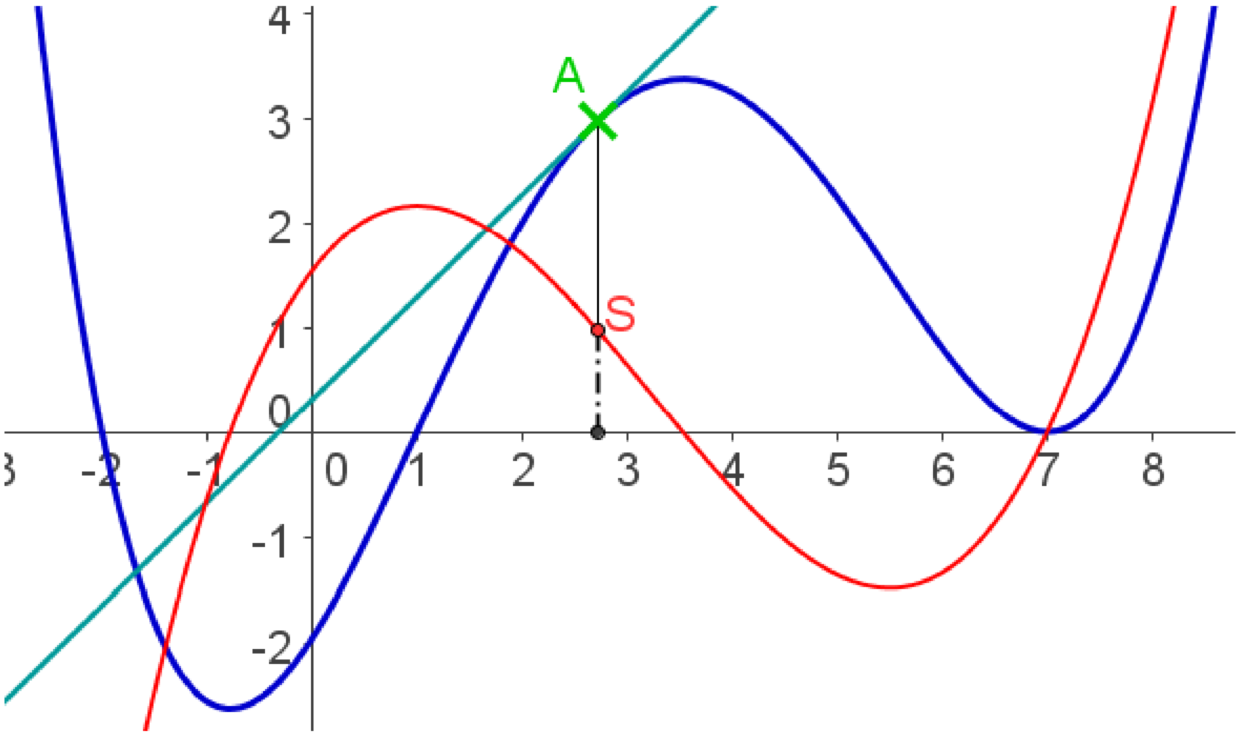
diff

The derivative f ‘ is the function, which shows the slope of the given function for every position x

Fahrrad pur

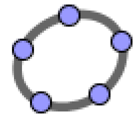
Fahrrad hier

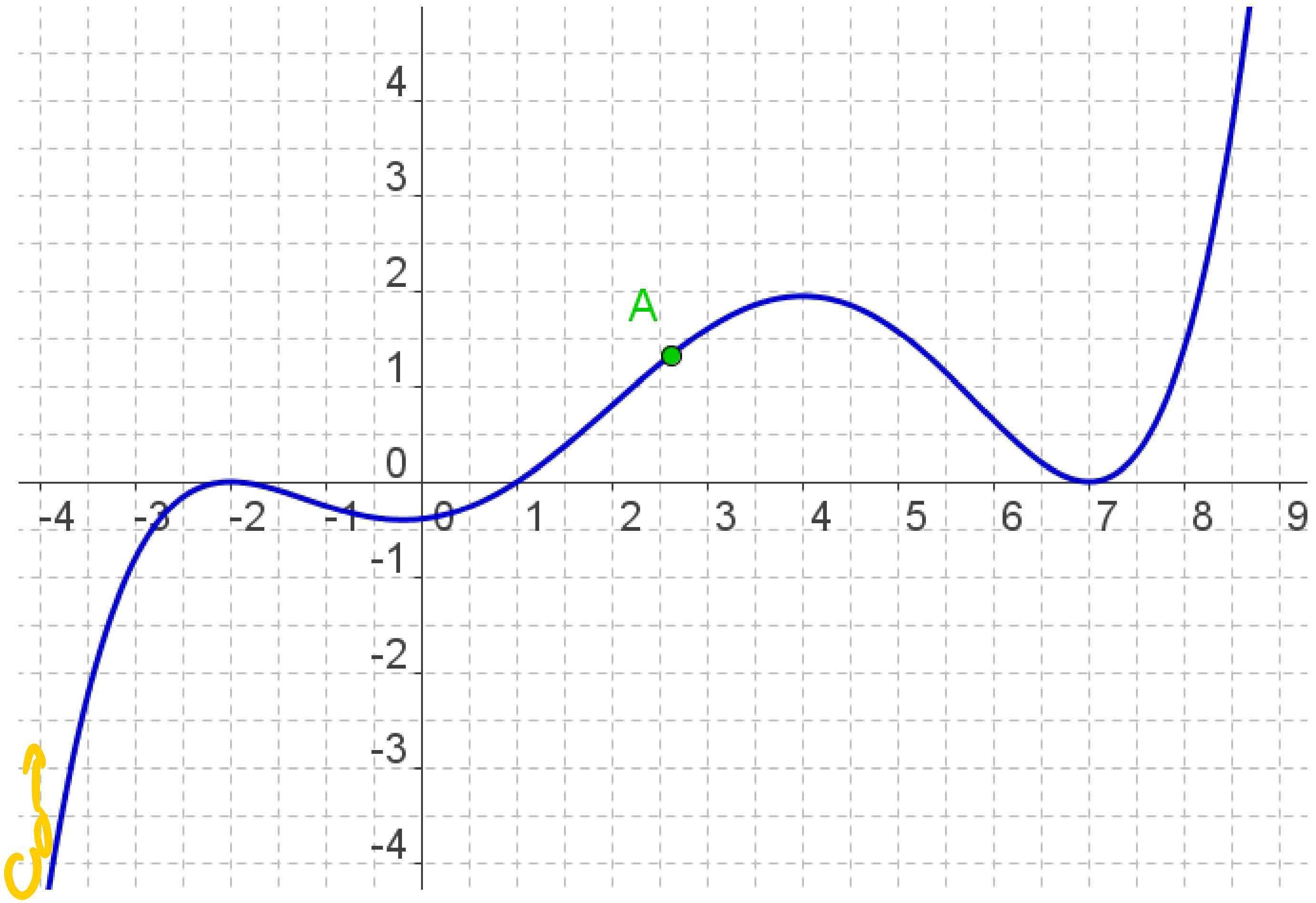
Fahrrad,



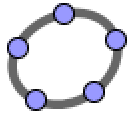
Now: the red function is the derivative of the blue function.

Übung 2 mit Funktionsgraphen

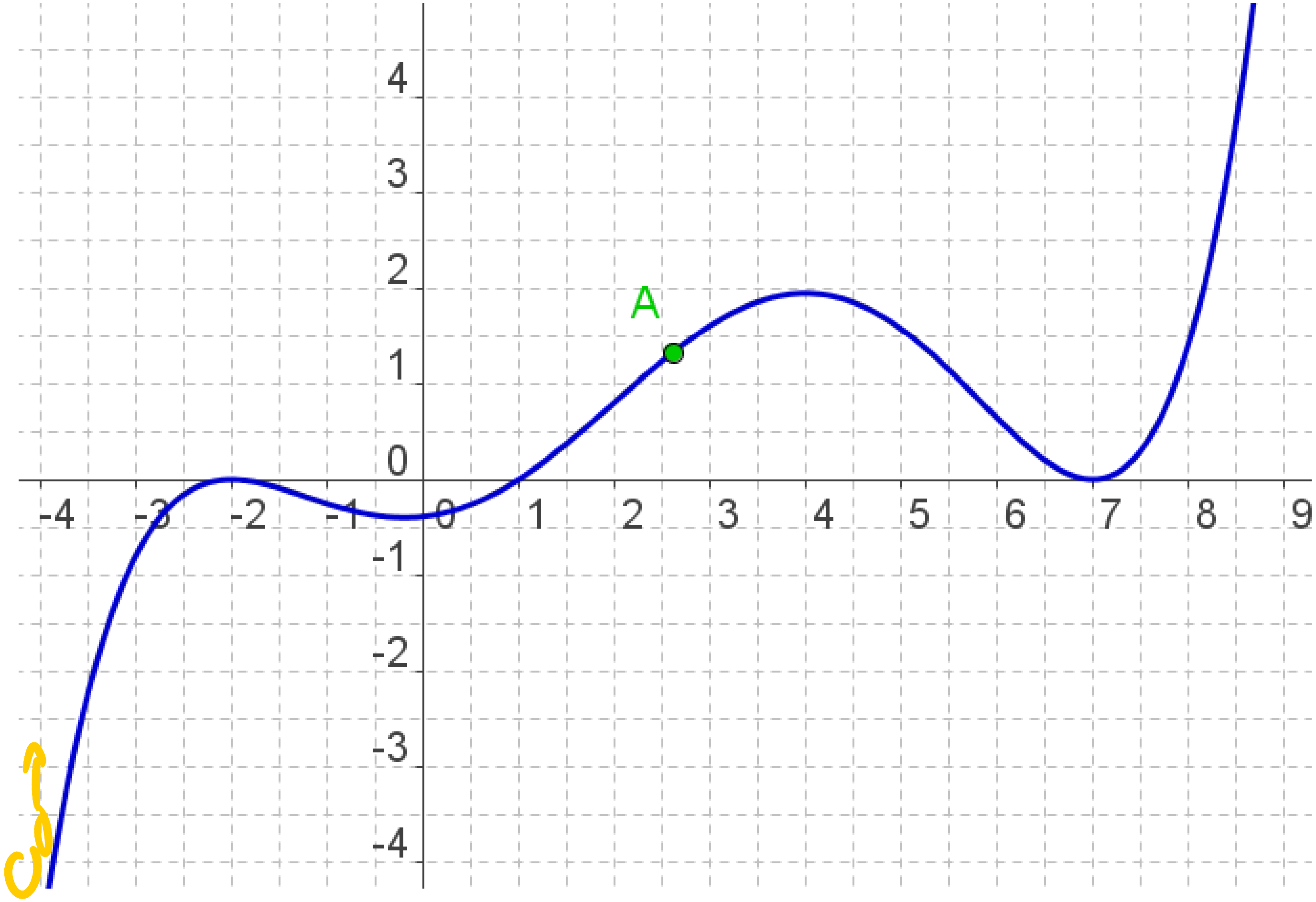




Practice 2 with Graphs of Functions

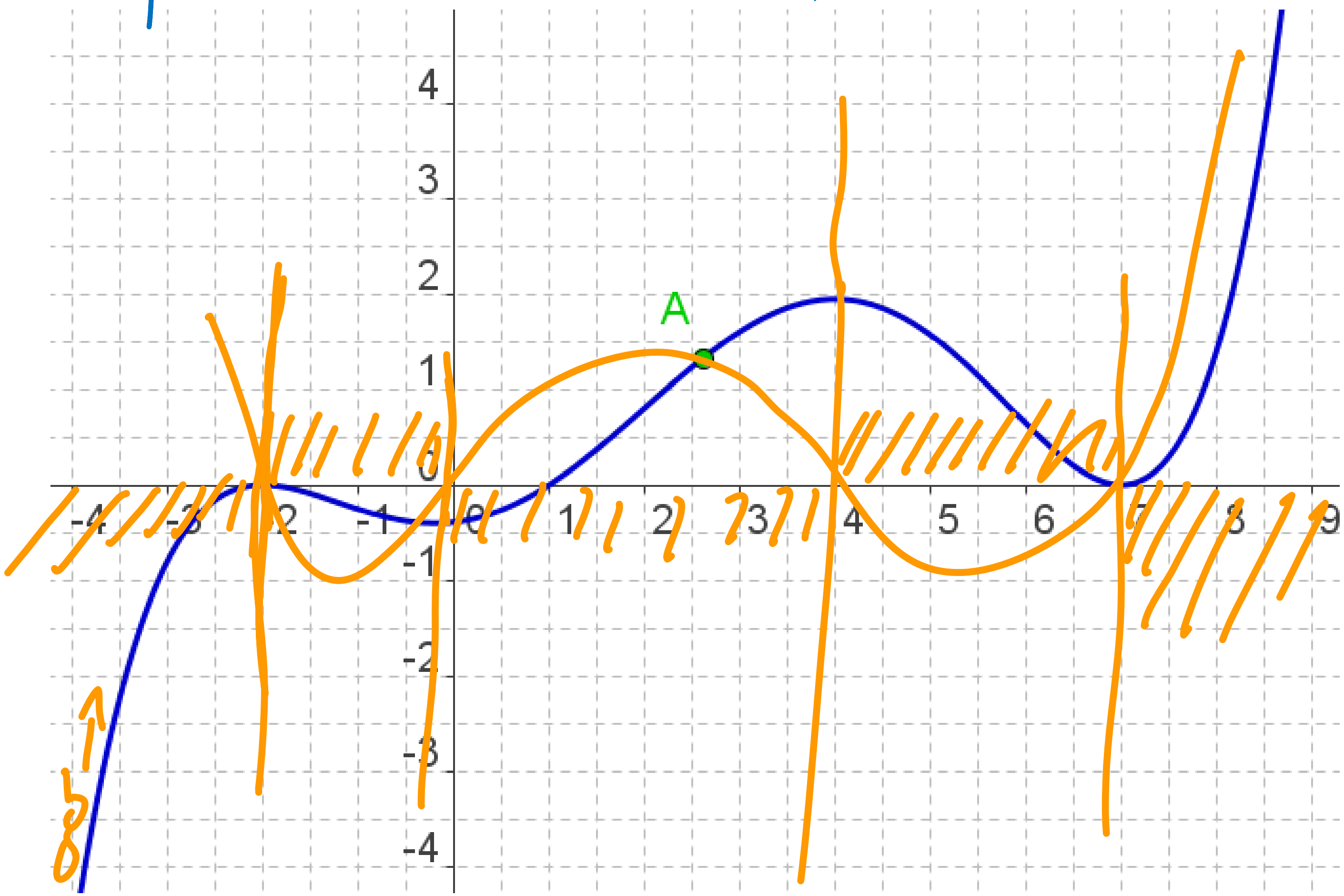


Fahrrad,
Bspl 2



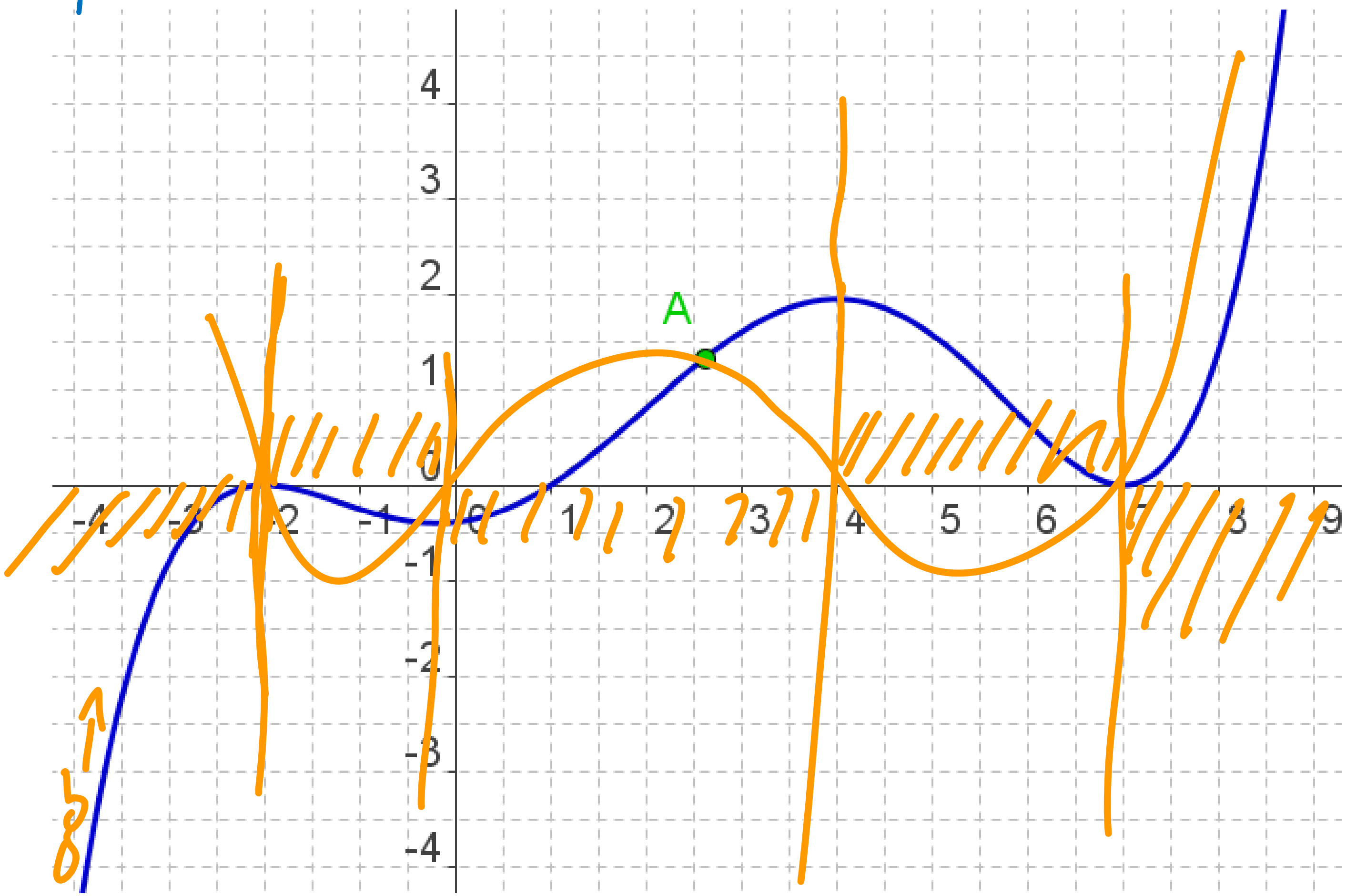
Übung 2 mit Funktionsgraphen

$f(x) = (x+2)^2 (x-1)(x-7)^2$

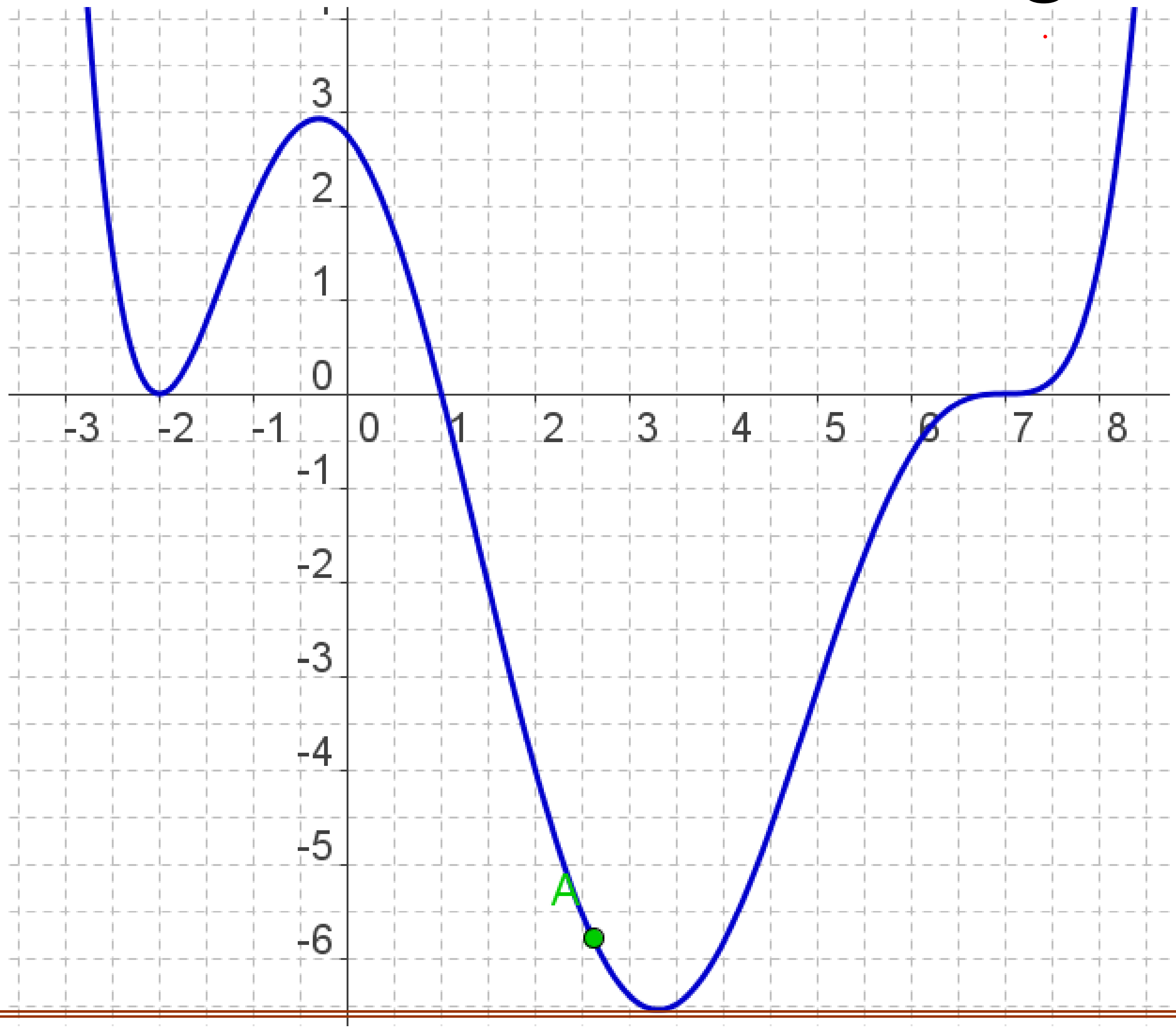


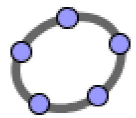
Practice 2 with Graphs of Functions

$f(x) = (x+2)^2 \cdot (x-1)(x-7)^2$



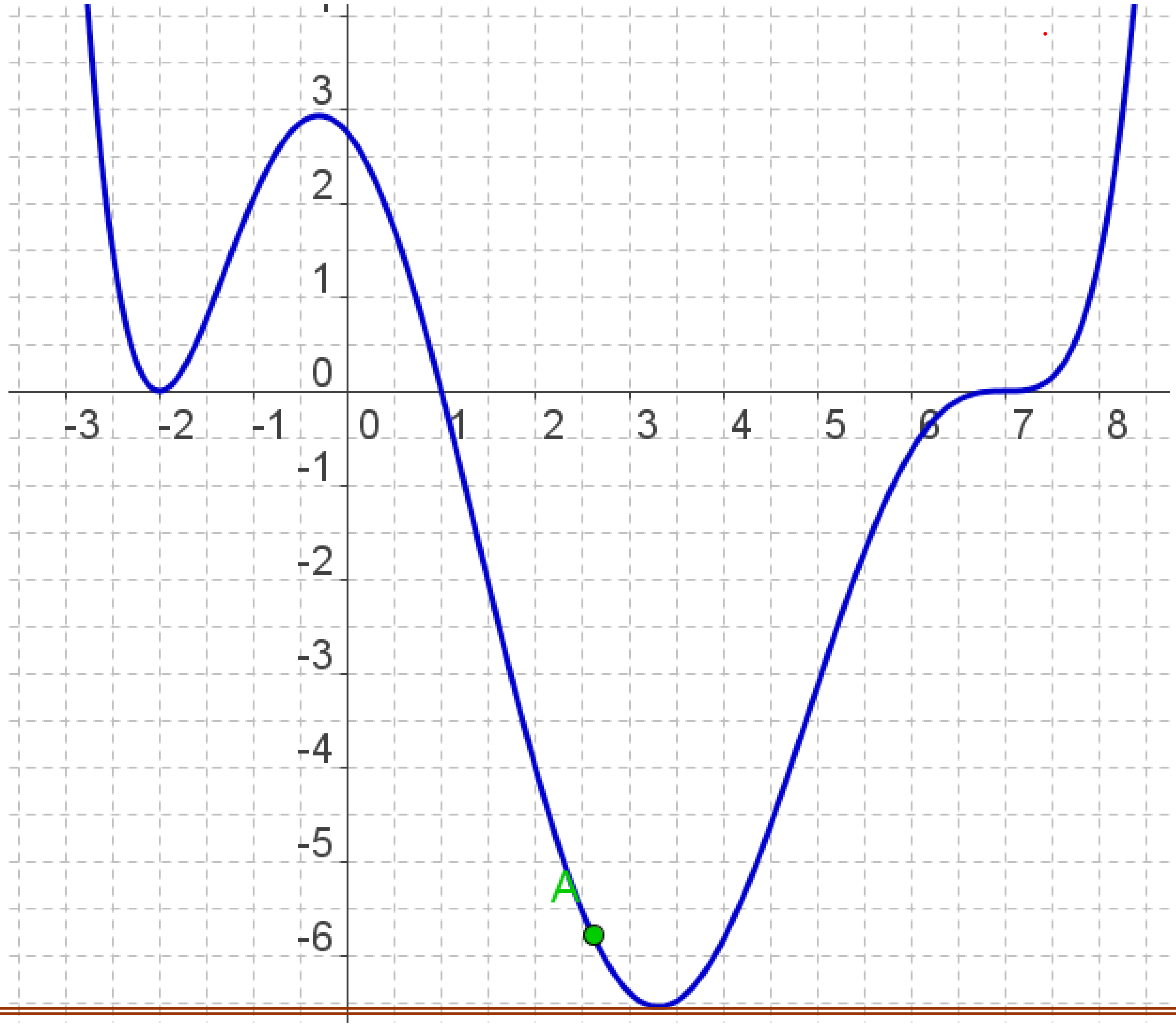
Übung 3 mit Funktionsgraphen und ihren Ableitungen

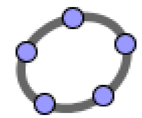




diff3

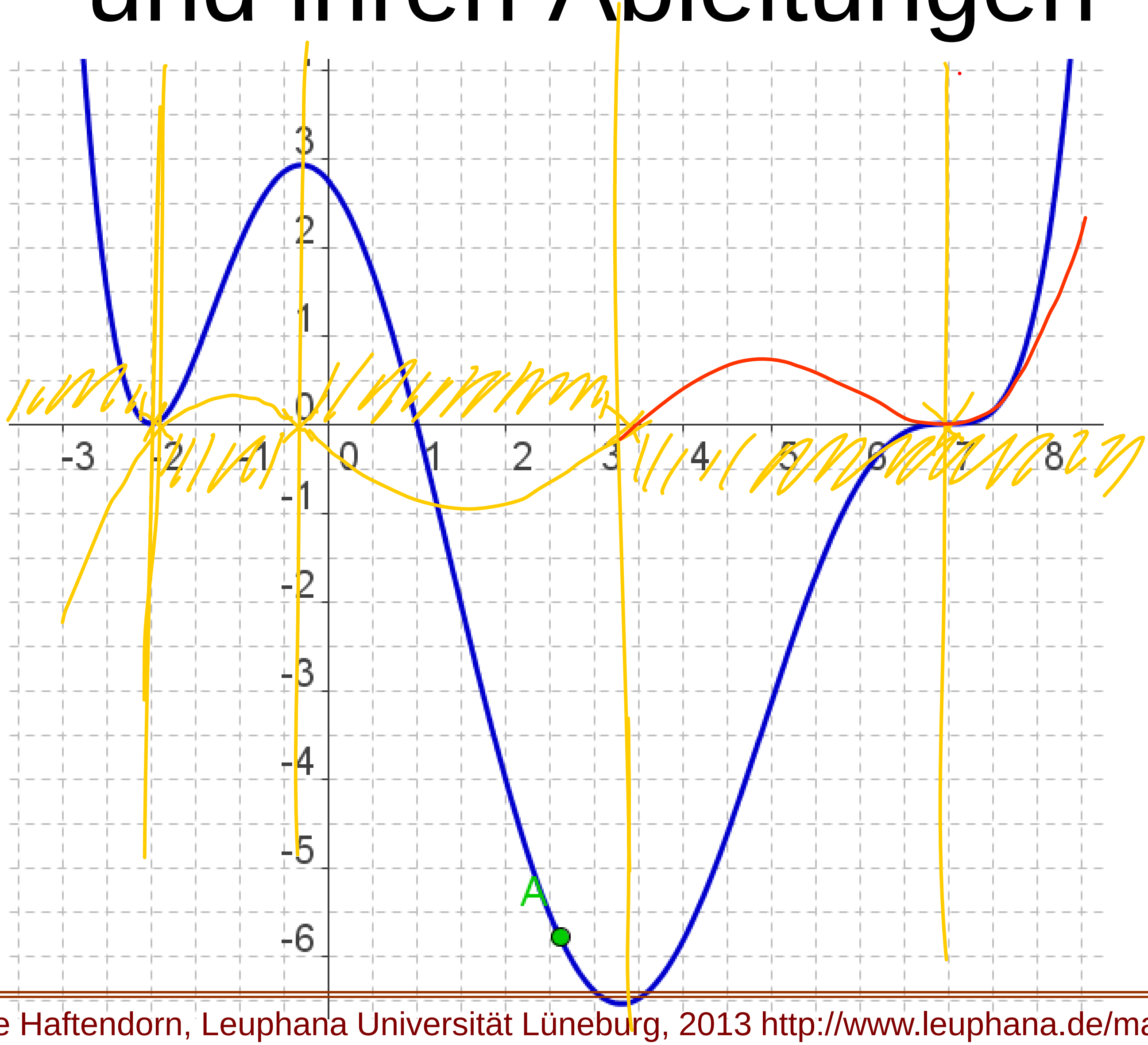
Practice 3 with Graphs of Functions and their Derivatives



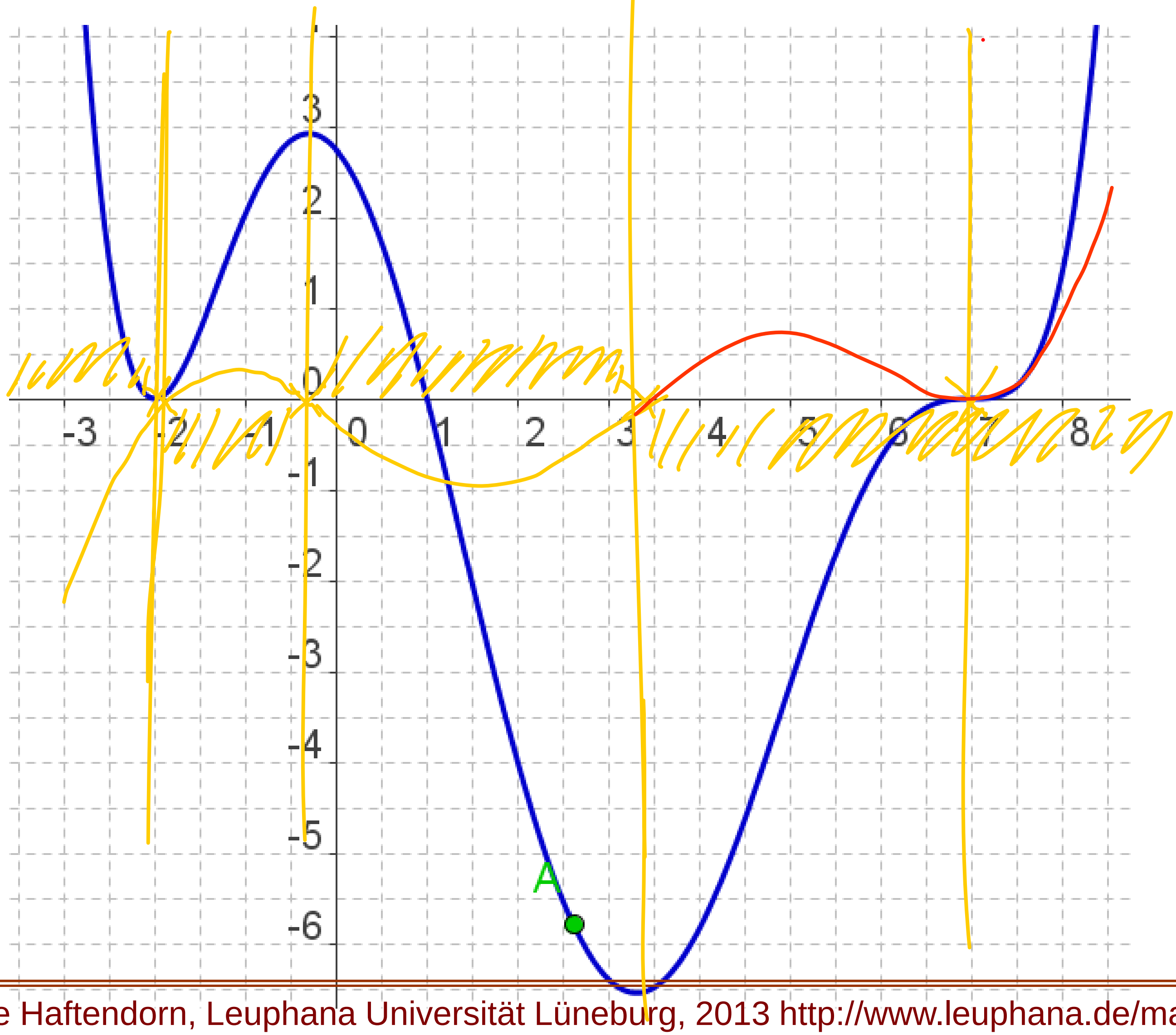


diff3

Übung 3 mit Funktionsgraphen und ihren Ableitungen



Practice 3 with Graphs of Functions and their Derivatives



e-Funktion, das ganze Geheimnis

Teil 1 Teil 2 Ableiten

$$f(x) = e^x$$

die e-Funktion ist diejenige Exponentialfunktion, die in (0/1) die Steigung 1 hat.

E-function, the Hole Mystery

Teil 1 Teil 2 Ableiten

$$f(x) = e^x$$

the one and only e-function is the exponential function who has in the point (0/1) the slope 1.

e-Funktion, das ganze Geheimnis

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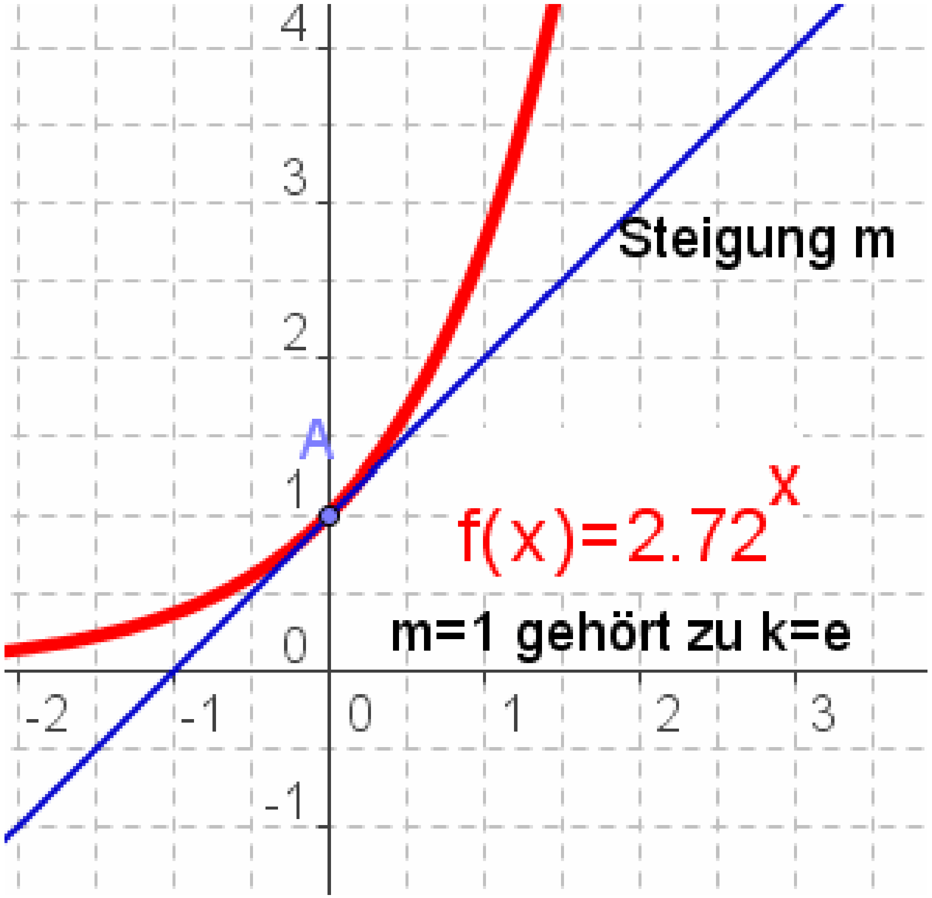


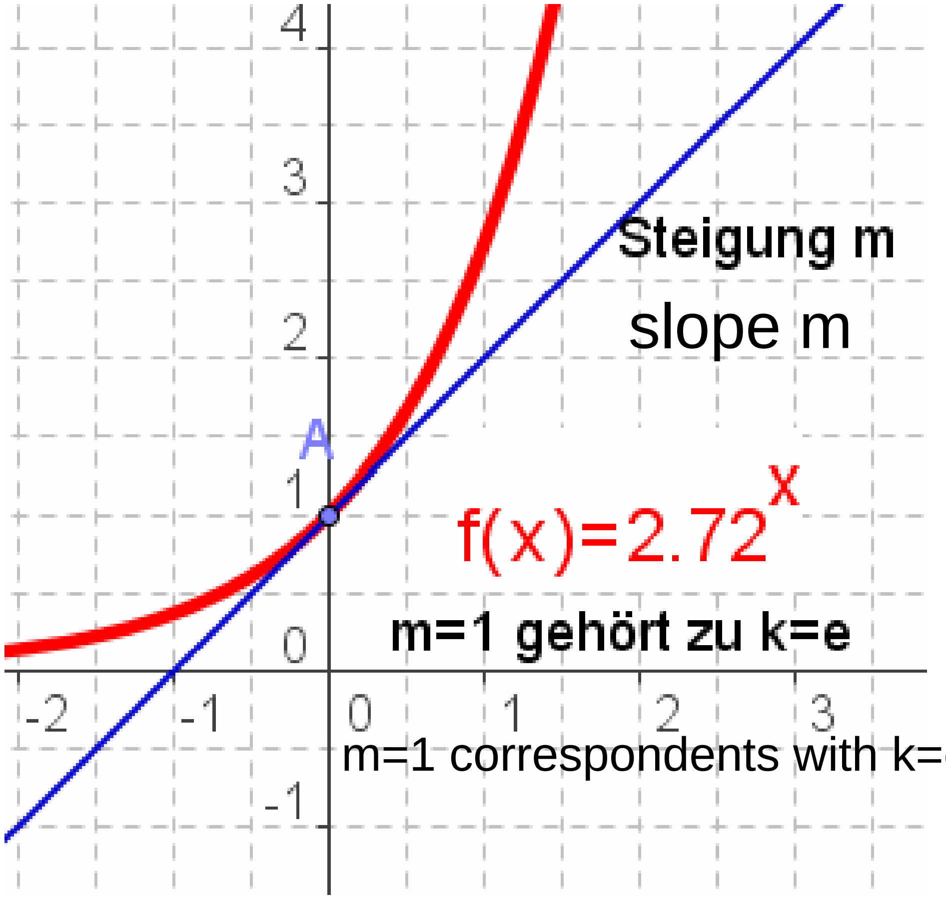
Die e-Funktion ist diejenige Funktion, die mit ihrer Ableitung übereinstimmt.

$$\left(e^x\right)' = e^x$$

E-function, the Hole Mystery

Teil 1 Teil 2 Ableiten

$$f(x) = e^x$$



m=1 correspondents with k=e

the one and only e-function is the exponential function who has in the point (0/1) the slope 1.

The e-function ist the only function who is identic with ist derivative.

$$\left(e^x\right)' = e^x$$