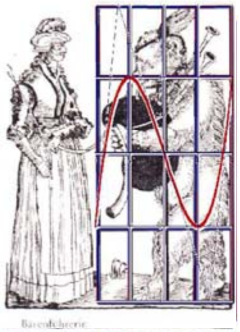
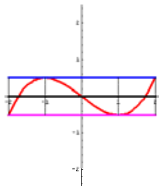
Polynome und mehrfache Nullstellen





Polynome sind Gefangene ihrer leicht durchschaubaren Eigenschaften.

Stichwort: Polynome im Affenkasten

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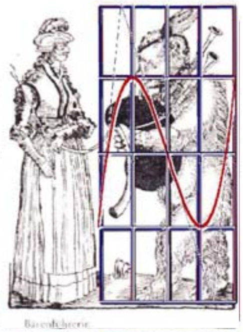
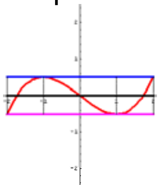
Polynomials and Multiple Zeros





polynomials are prisons of their obvious characteristics

These are named: polynomials in an „arp box“

key word: Polynome im Affenkasten

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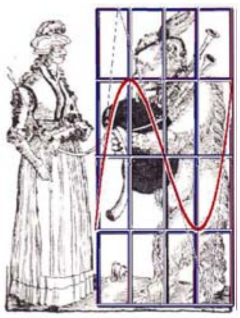
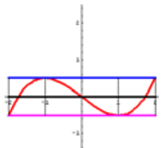
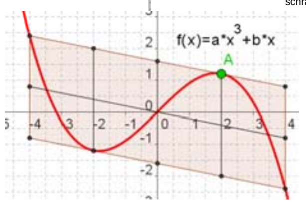
Polynome und mehrfache Nullstellen





gerade

schräg



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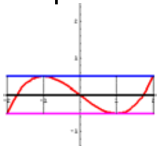
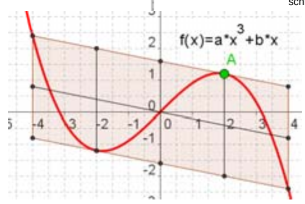
Polynomials and Multiple Zeros



gerade

schräg



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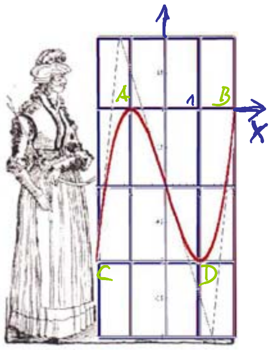
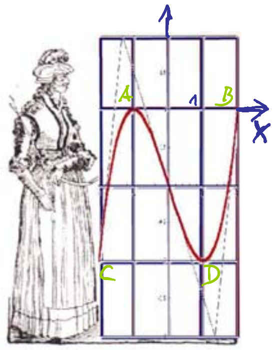
Nullstellen ⟺ Linearfaktoren



$$f(x) = t\,(x+1)^2\,\overbrace{(x-2)}^{Lin.}$$

double zero	single zero
x= -1	x= 2
A	B

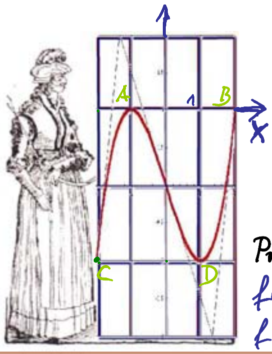
Zeros ⟺ Linear Factors



$$f(x) = t\,(x+1)^2\,\overbrace{(x-2)}^{Lin.}$$

doppelte Nullstelle	einfache Nullstelle
x= -1	x= 2
A	B

Nullstellen ⟺ Linearfaktoren



$f(x) = (x+1)^2(x-2)$

doppelte Nullstelle — x= -1 — A

einfache Nullstelle — x= 2 — B

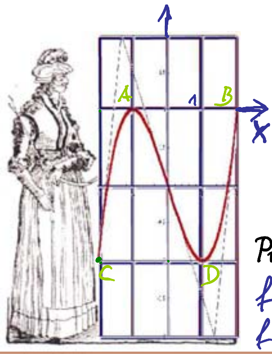
Proben

$f(-2) = (-2+1)^2(-2-2) \;\; C$
$\quad = -4$"

$f(1) = (1+1)^2(1-2) = -4 \;\; D$

7

Zeros ⟺ Linear Factors



$f(x) = (x+1)^2(x-2)$

double zero — x= -1 — A

single zero — x= 2 — B

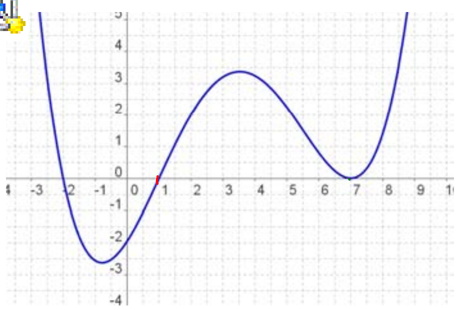
Proben

$f(-2) = (-2+1)^2(-2-2) \;\; C$
$\quad = -4$"

$f(1) = (1+1)^2(1-2) = -4 \;\; D$

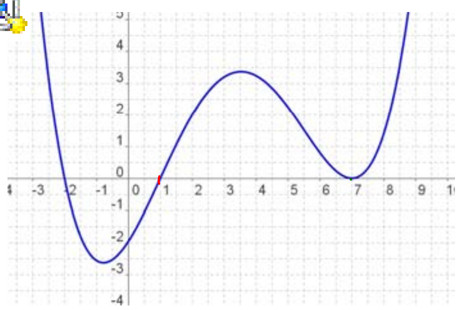
8

Welche Gleichung kann dieses Polynom haben?



9

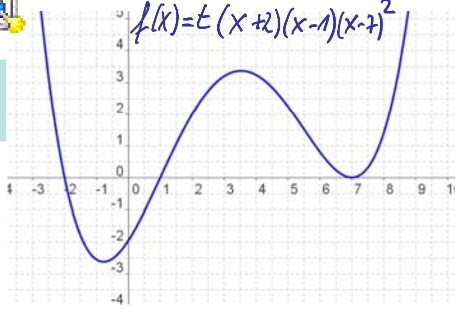
Which Equation ist Possible for this Polynomial?



10

Welche Gleichung kann dieses Polynom haben?

$f(x) = t\,(x+2)(x-1)(x-7)^2$



Diese Funktion

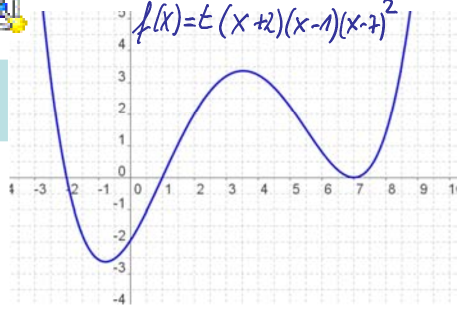
Vieta

Vieta, mehr

11

Which Equation ist Possible for this Polynomial?

$f(x) = t\,(x+2)(x-1)(x-7)^2$



this function

Vieta

Vieta, plus

12

Was ist eigentlich ein Polynom?

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

Ein Polynom ist eine Summe von Potenzfunktionen.
Der höchste Exponent, der vorkommt, heißt Grad des Polynoms.

- Polynome 1. Grades sind die Geraden
- Polynome 2. Grades sind die Parabeln
- Polynome 3. Grades haben immer eine symmetrische s-Form.
- Polynome 4. Grades haben höchstens 3 Extrema.
- Je höher der Grad, desto vielfältigere Formen sind möglich.

How is a Polynomial Defined?

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

A polynomial is a sum of power functions.
The highest exponent is named **degree of the polynomial**.

- Polynomials of degree 1 are the straight lines.
- Polynomials of degree 2 are the parabolas.
- Polynomials of degree always have a symmetrical s-Form.
- Polynomials of degree 4 have at most 3 extrema.
- The higher the degree the manifold forms are possible.

Polynome und ihre Linearfaktoren

Jede reelle Nullstelle a
erzeugt einen Linearfaktor. $(x-a)$

$$f(x) = (x-a)\, q(x)$$

Wenn das Restpolynom auch noch die Nullstelle a
enthält, kann man den Linearfaktor mehrfach „herausziehen".

$$f(x) = (x-a)^k\, p(x) \quad mit \quad p(a) \neq 0$$

Geht das maximal k-mal, dann heißt a k-fache Nullstelle,
oder „Nullstelle der Vielfachheit k"

Polynomials and their Linear Factors

Every real zero a
corresponds to a linear factor. $(x-a)$

$$f(x) = (x-a)\, q(x)$$

If the remaining polynomial has zero a too,
you can „pull out" the linear factor twice ore more.

$$f(x) = (x-a)^k\, p(x) \quad mit \quad p(a) \neq 0$$

a is named **zero of degree k**, if this is possible k times at most,
An othe name ist „**zero of the order k**".

Polynome und ihre Linearfaktoren

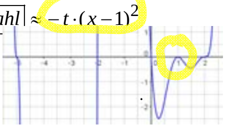
$$f(x) = (x-a)^k\, p(x) \quad mit \quad p(a) \neq 0$$

In der Nähe eine k-fachen Nullstelle verhält sich das Polynom
wie sich die k-Potenzfunktion im Ursprung verhält.

$f(x) = (x+5)^3 (x+2)\; x\; (x-1)^2 (x-2)^3$

$f(nahe\ 1) = \boxed{pos.Zahl} \cdot (x-1)^2 \cdot \boxed{neg.Zahl} \approx -t \cdot (x-1)^2$

Grad 10 Gesamtverlauf



Ein Polynom n-ten Grades hat höchsten n Nullstellen, mit ihrer Vielfachheit gezählt. Fundamentalsatz der Algebra (reell)

Polynomials and their Linear Factors

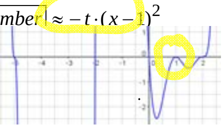
$$f(x) = (x-a)^k\, p(x) \quad mit \quad p(a) \neq 0$$

The polynomial is in the neighbourhood of k-order zero similar
to a power function of degree k near the origin.

$f(x) = (x+5)^3 (x+2)\; x\; (x-1)^2 (x-2)^3$

$f(near\ 1) = \boxed{pos.number} \cdot (x-1)^2 \cdot \boxed{neg.number} \approx -t \cdot (x-1)^2$

degree 10 outside form



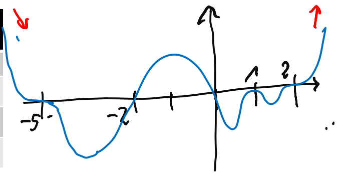
A polynomial of degree n has at most n zeros, counted with their orders. fundamental theorem of algebra (real version)

Polynome und ihre Linearfaktoren

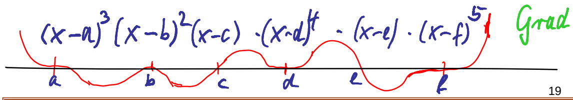
$$f(x) = (x+5)^3(x+2)\ x\ (x-1)^2\ (x-2)^3$$

Qualitativer Graph eines durch Linearfaktoren gegeben Polynoms

Vorzei-chen	Grad	Ges. Verlauf
+	gerade	↘ ↗
−	gerade	↗ ↘
+	ungerade	↗ ↗
−	ungerade	↘ ↘



$(x-a)^3(x-b)^2(x-c)\cdot(x-d)^4\cdot(x-e)\cdot(x-f)^5$ Grad?

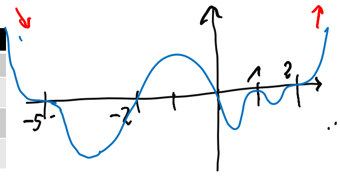


Polynomials and their Linear Factors

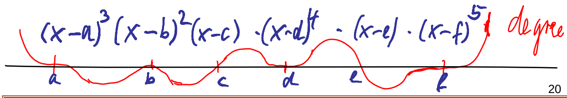
$$f(x) = (x+5)^3(x+2)\ x\ (x-1)^2\ (x-2)^3$$

Qualitative Graph of a polynomial, given with its linear factors.

sign	degree	outer form
+	even	↘ ↗
−	even	↗ ↘
+	odd	↗ ↗
−	odd	↘ ↘

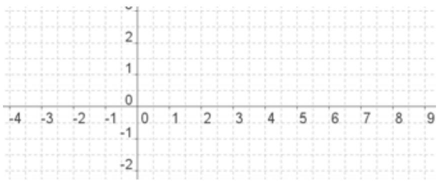


$(x-a)^3(x-b)^2(x-c)\cdot(x-d)^4\cdot(x-e)\cdot(x-f)^5$ degree?



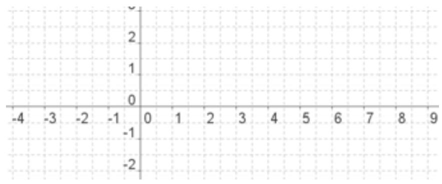
Übung 2 mit Polynomen

$$f(x)=(x+2)^2\cdot(x-1)(x-7)^2$$



Practice 2 with Polynomials

$$f(x)=(x+2)^2\cdot(x-1)(x-7)^2$$



Übung 2 mit Polynomen

$$f(x)=(x+2)^2\cdot(x-1)(x-7)^2$$

Grad 5

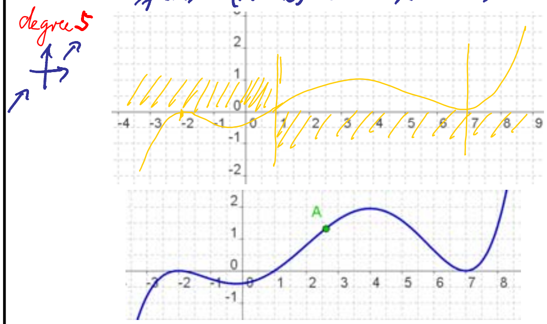
+ ↗

Practice 2 with Polynomials

$$f(x)=(x+2)^2\cdot(x-1)(x-7)^2$$

degree 5

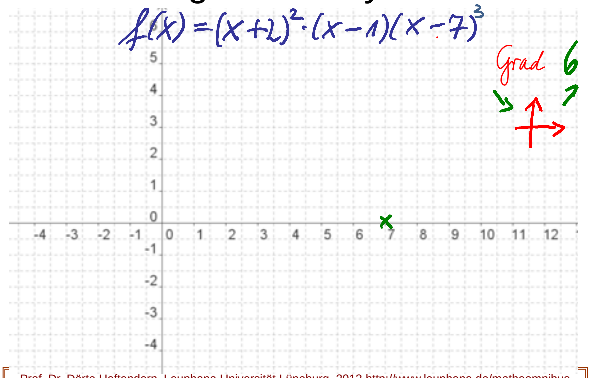
+ ↗



Übung 3 mit Polynomen

$f(x) = (x+2)^2 \cdot (x-1)(x-7)^3$

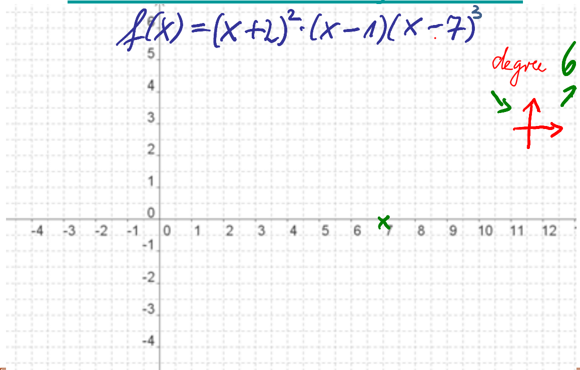
Grad 6



Practice 3 with Polynomials

$f(x) = (x+2)^2 \cdot (x-1)(x-7)^3$

degree 6



Übung 3 mit Polynomen

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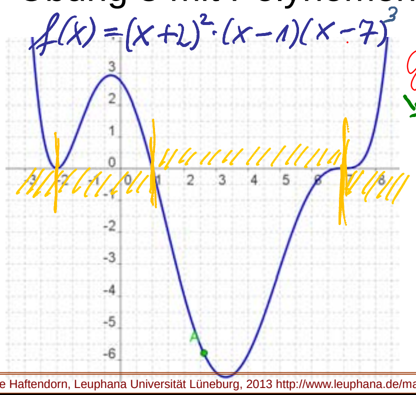
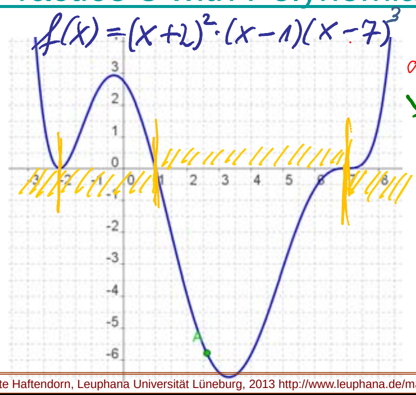
Grad 6



Practice 3 with Polynomials

$f(x) = (x+2)^2 \cdot (x-1)(x-7)^3$

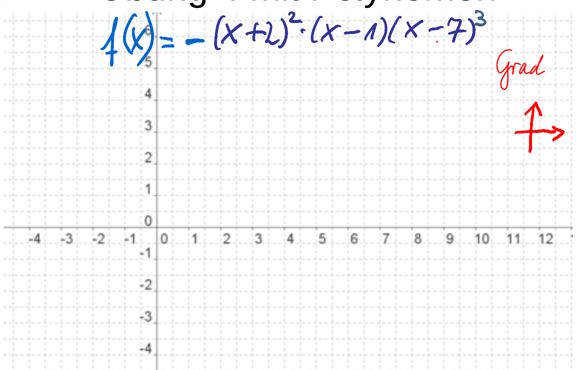
degree 6



Übung 4 mit Polynomen

$f(x) = -(x+2)^2 \cdot (x-1)(x-7)^3$

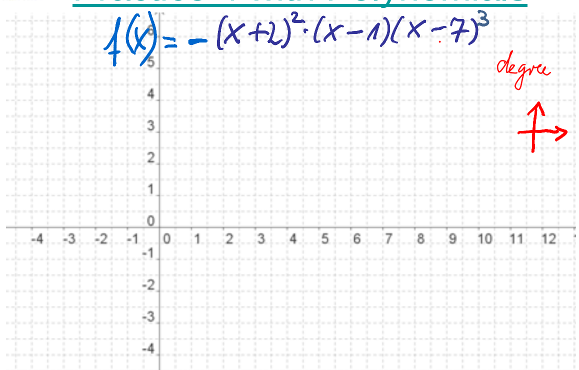
Grad



Practice 4 with Polynomials

$f(x) = -(x+2)^2 \cdot (x-1)(x-7)^3$

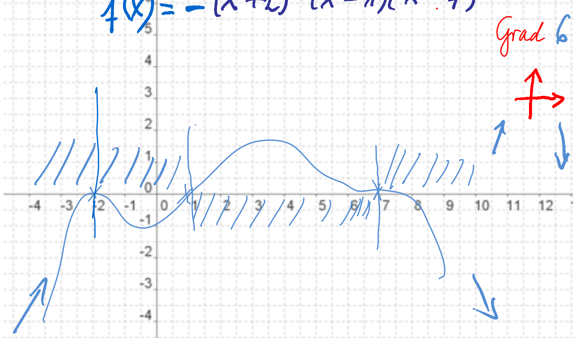
degree



Übung 4 mit Polynomen

$f(x) = -(x+2)^2 \cdot (x-1)(x-7)^3$

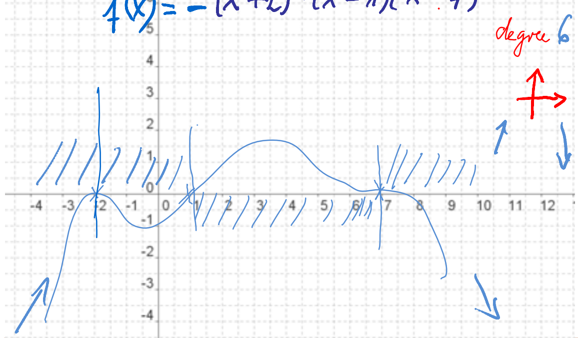
Grad 6



Practice 4 with Polynomials

$f(x) = -(x+2)^2 \cdot (x-1)(x-7)^3$

degree 6



Funktionen als zentrales Werkzeug

- Potenzfunktionen
- Polynome
- **Trigonometrische Funktionen**
- **Exponentialfunktionen**
- **Davon so manche Umkehrfunktionen**
  - Wurzelfunktionen
  - Arkusfunktionen
  - **Logarithmusfunktionen**

Das war's dann aber auch

Und das noch koppeln mit $+ \ - \ \cdot \ / \ \wedge$ und Verkettung.

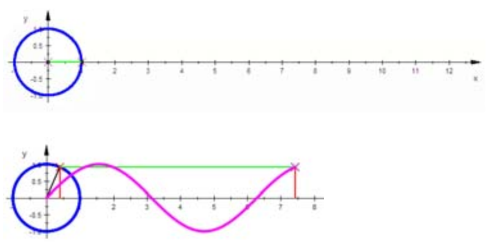
Functions as a Central Tool

- power functions
- polynomials
- **trigonometric functions**
- **exponential functions**
- **some inverse functions**
  - root functions
  - arc functions
  - **logarithmic functions**

that's all for normal purpose

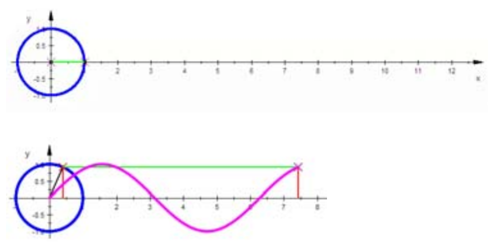
and at that you can caculate $+ \ - \ \cdot \ / \ \wedge$ and build chains.

Funktionen als zentrales Werkzeug



Sinusfunktion

Functions as a Central Tool

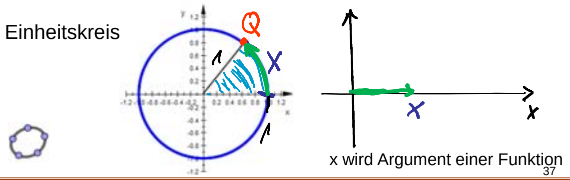


sine function

Die Winkel-Funktionen

Der Punkt Q läuft im Einheitskreis vom Start (1/0).
(mathematisch positiv = gegen die Uhr)

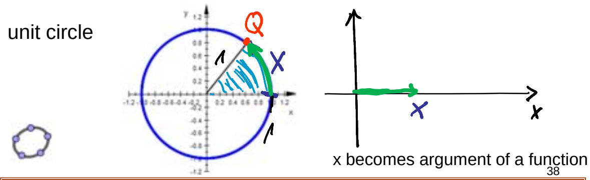
Den von Q zurückgelegten Weg x nennt man auch „das Bogenmaß des Winkels“, um den sich Q gedreht hat.
Kurz: x ist der Winkel im Bogenmaß



Angle Functions

Point Q goes in the unit circle with start in (1/0).
(mathematical positiv = against the clock)

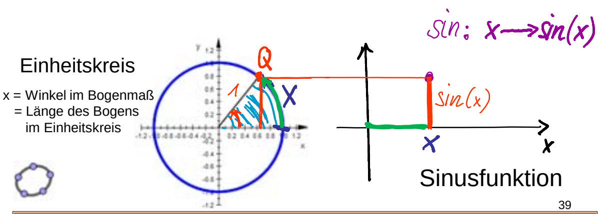
The distance x, covered by Q on the circle, ist named „the radian measure of the angle“, the angle of rotation of Q.
short: x is the angle in radian measure



Die Sinus-Funktion

Dem Winkel x wird nun die Ordinate von Q zugeordnet.

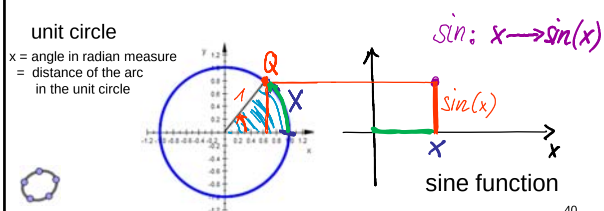
Die Funktion, die das leistet, heißt Sinus-Funktion.



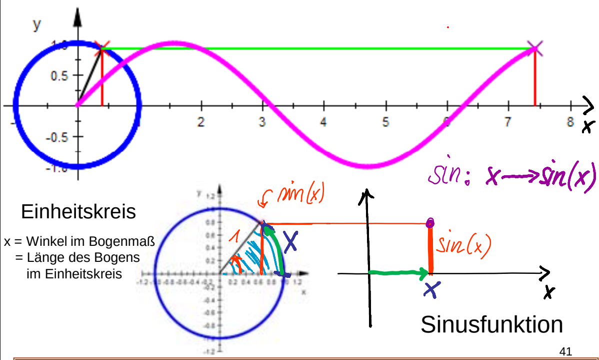
Sine Function

The angle x correlates with the ordinate of Q.

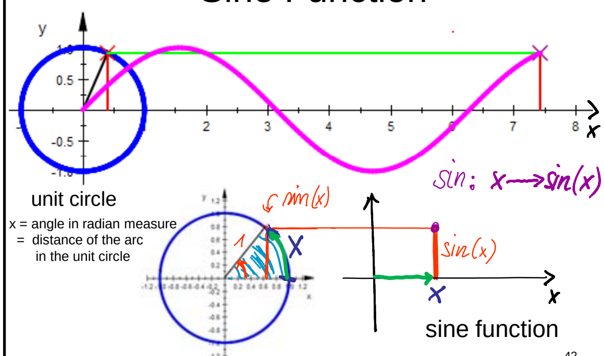
The function, which is able to do do, is the sine-function.



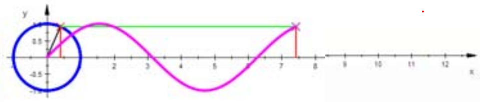
Die Sinus-Funktion



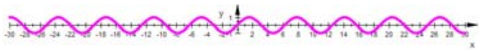
Sine Function



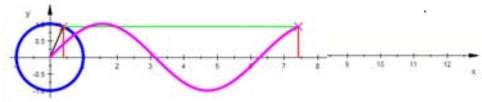
Eigenschaften der Sinus-Funktion



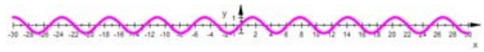
- Die Sinus-Funktion ist periodisch.
- Die Periode ist 2π .
- Die Sinuswerte liegen zwischen -1 und + 1.
- Die Sinusbögen sind symmetrisch.
- Die Sinuskurve ist punktsymmetrisch zum Ursprung



Properties of the Sine Function



- The sine function is periodic.
- The period is 2π .
- The sine ordinates are between -1 and + 1.
- The arcs of the sine function are symmetric.
- The sine curve has the origin as a point of symmetry.



Die Sinus-Funktion

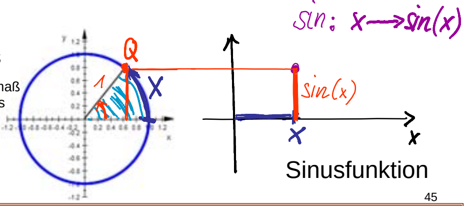
Dem Winkel x wird nun die Ordinate von Q zugeordnet.

Die Funktion, die das leistet, heißt Sinus-Funktion.

sin: $x \longrightarrow \sin(x)$

Einheitskreis

x = Winkel im Bogenmaß
= Länge des Bogens
im Einheitskreis



Sinusfunktion

Sine Function

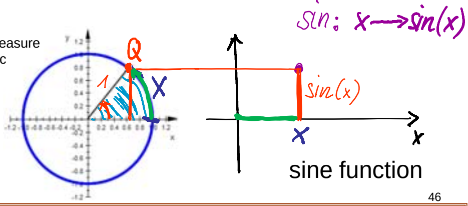
The angle x correlates with the ordinate of Q.

The function, which is able to do do, is the sine function.

sin: $x \longrightarrow \sin(x)$

unit circle

x = angle in radian measure
= distance of the arc
in the unit circle



sine function

Die Kosinus-Funktion

Dem Winkel x wird nun die ~~Ordinate~~ Abszisse von Q zugeordnet.

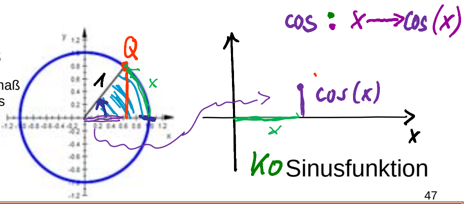
Die Funktion, die das leistet, heißt ~~Sinus~~ Kosinus-Funktion.

cos : $x \longrightarrow \cos(x)$

Einheitskreis

x = Winkel im Bogenmaß
= Länge des Bogens
im Einheitskreis

Tangens Kosinus



Ko Sinusfunktion

Cosine Function

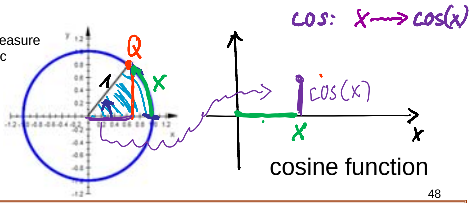
The angle x correlates with the ~~ordinate~~ abscissa of Q.

The function, which is able to do do, is the ~~sine~~ cosine function.

cos: $x \longrightarrow \cos(x)$

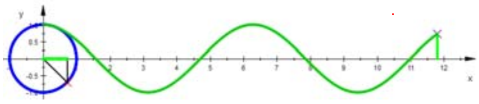
unit circle

x = angle in radian measure
= distance of the arc
in the unit circle

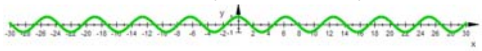


cosine function

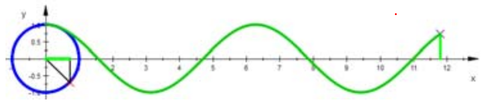
Eigenschaften der Kosinus-Funktion



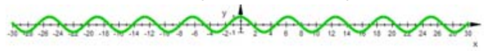
- Die Kosinus-Funktion ist periodisch.
- Die Periode ist 2π .
- Die Kosinuswerte liegen zwischen -1 und + 1.
- Die Kosinusbögen sind symmetrisch.
- Die Kosinuskurve ist symmetrisch zur y-Achse



Properties of the Cosine Function



- The cosine function is periodic.
- The period is 2π .
- The ordinates of the cosine are between -1 and + 1.
- The arcs of the cosine sind symmetrisch.
- The cosine curve is symmetric to the y-axis

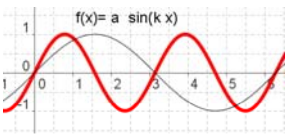


Sinus strecken und stauchen

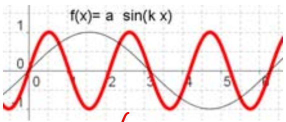
To Stretch and Compress the Sine Function

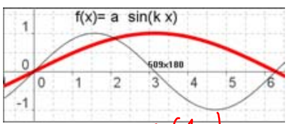
Funktionen strecken und stauchen

$y = \sin(2x)$



Ein Faktor direkt beim x sorgt für waagerechtes Strecken und Stauchen.

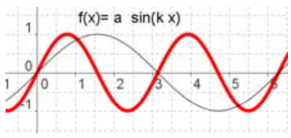




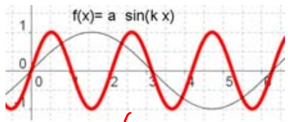
$y = \sin(3x)$ $y = \sin(\frac{1}{2}x)$

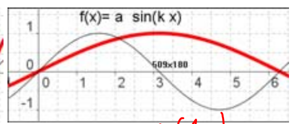
To Stretch and Compress the Functions

$y = \sin(2x)$



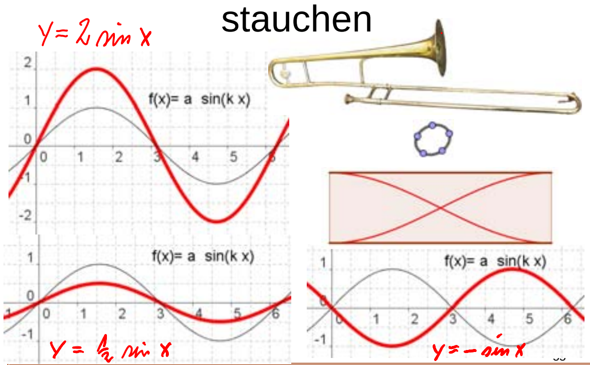
to stretch and compress in the direction of the x-axis you must put a factor to x directly



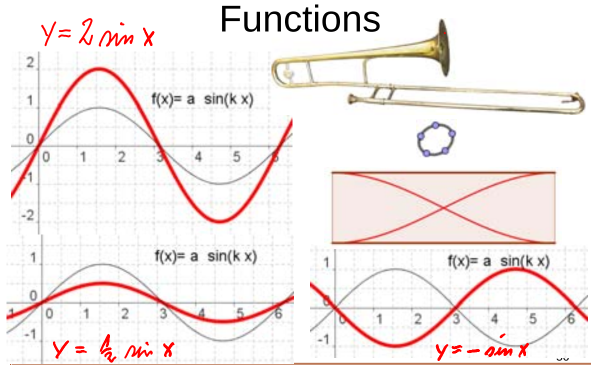


$y = \sin(3x)$ $y = \sin(\frac{1}{2}x)$

Funktionen strecken und stauchen



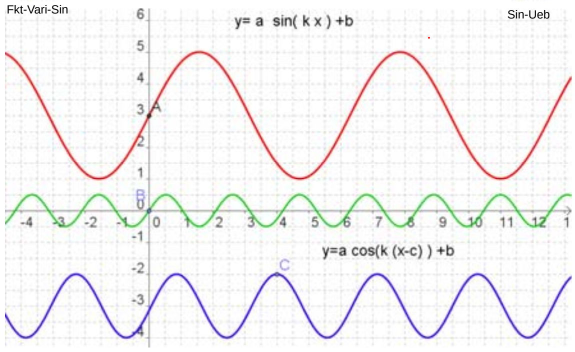
To Stretch and Compress the Functions



Funktionen variieren

Fkt-Vari-Sin

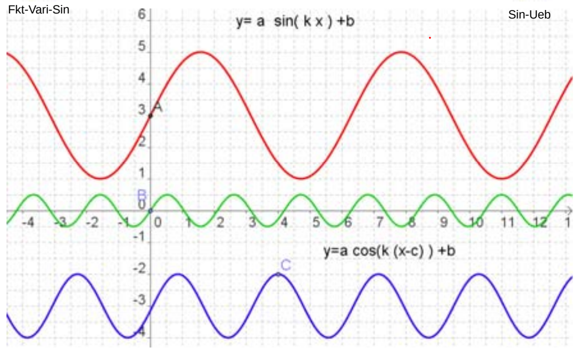
Sin-Ueb



Variation of Functions

Fkt-Vari-Sin

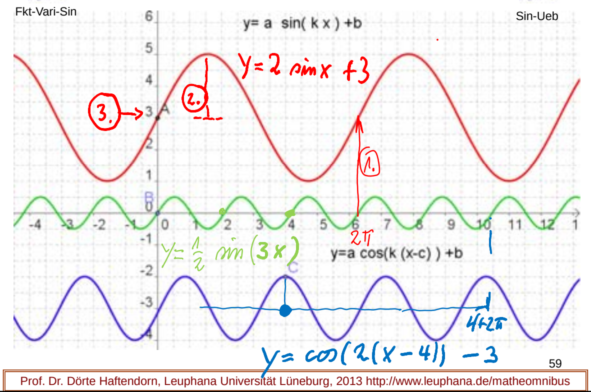
Sin-Ueb



Funktionen variieren

Fkt-Vari-Sin

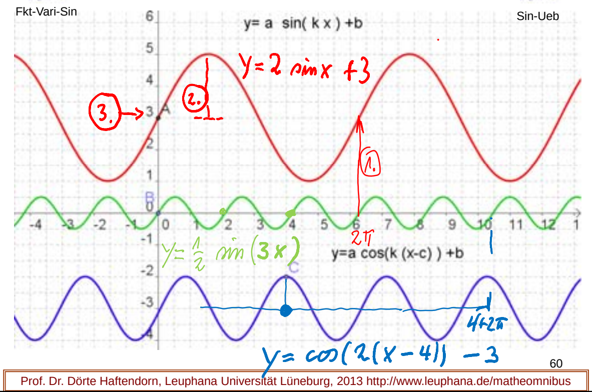
Sin-Ueb



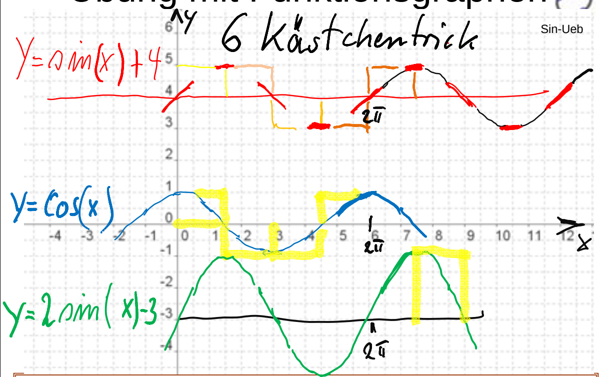
Variation of Functions

Fkt-Vari-Sin

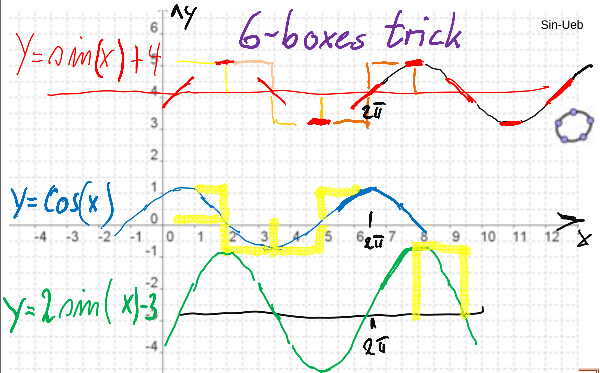
Sin-Ueb



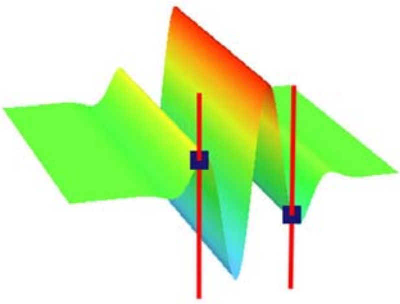
Übung mit Funktionsgraphen



How to Sketch Sine and Cosine Function



Welle, wave



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