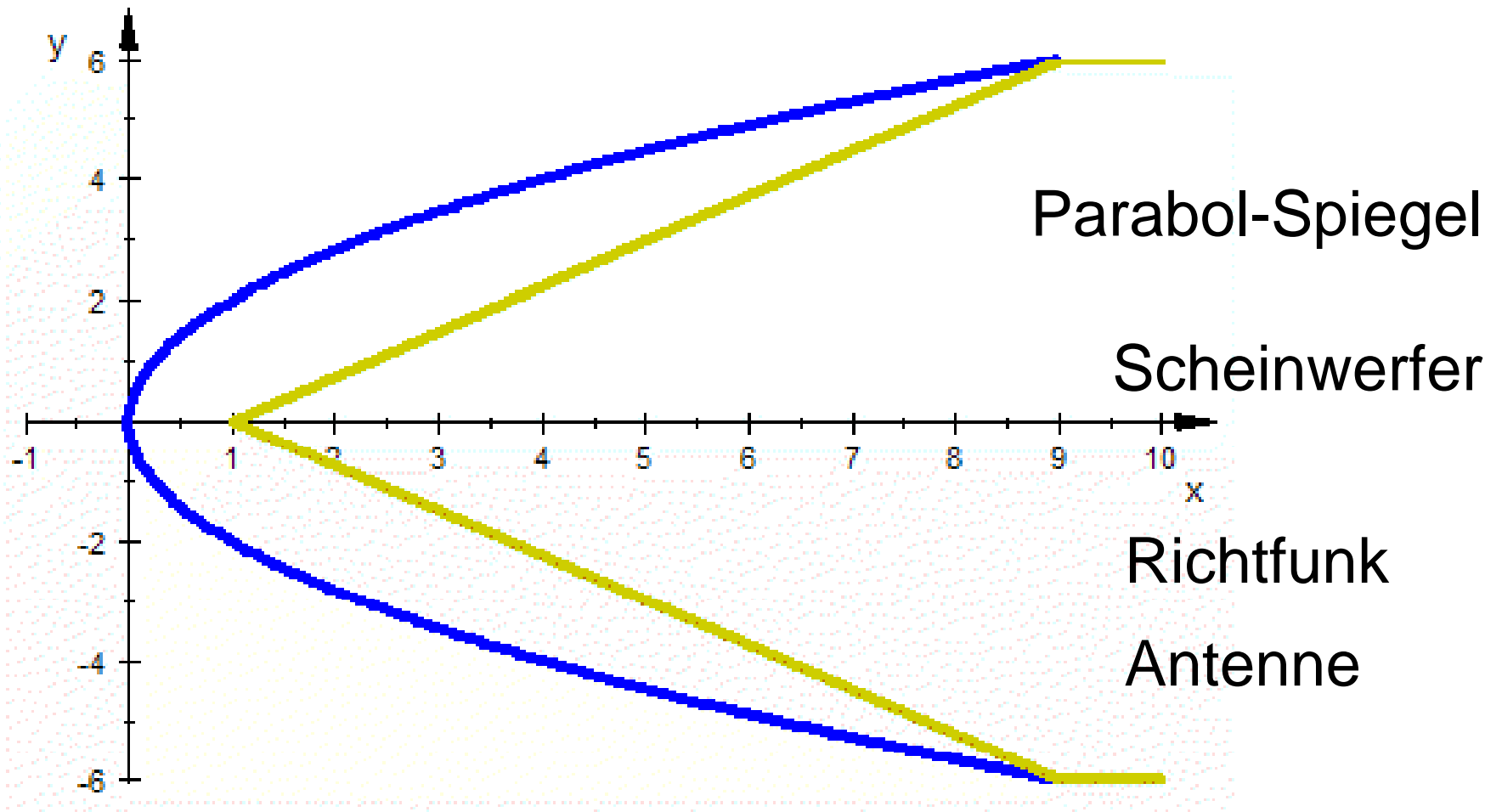
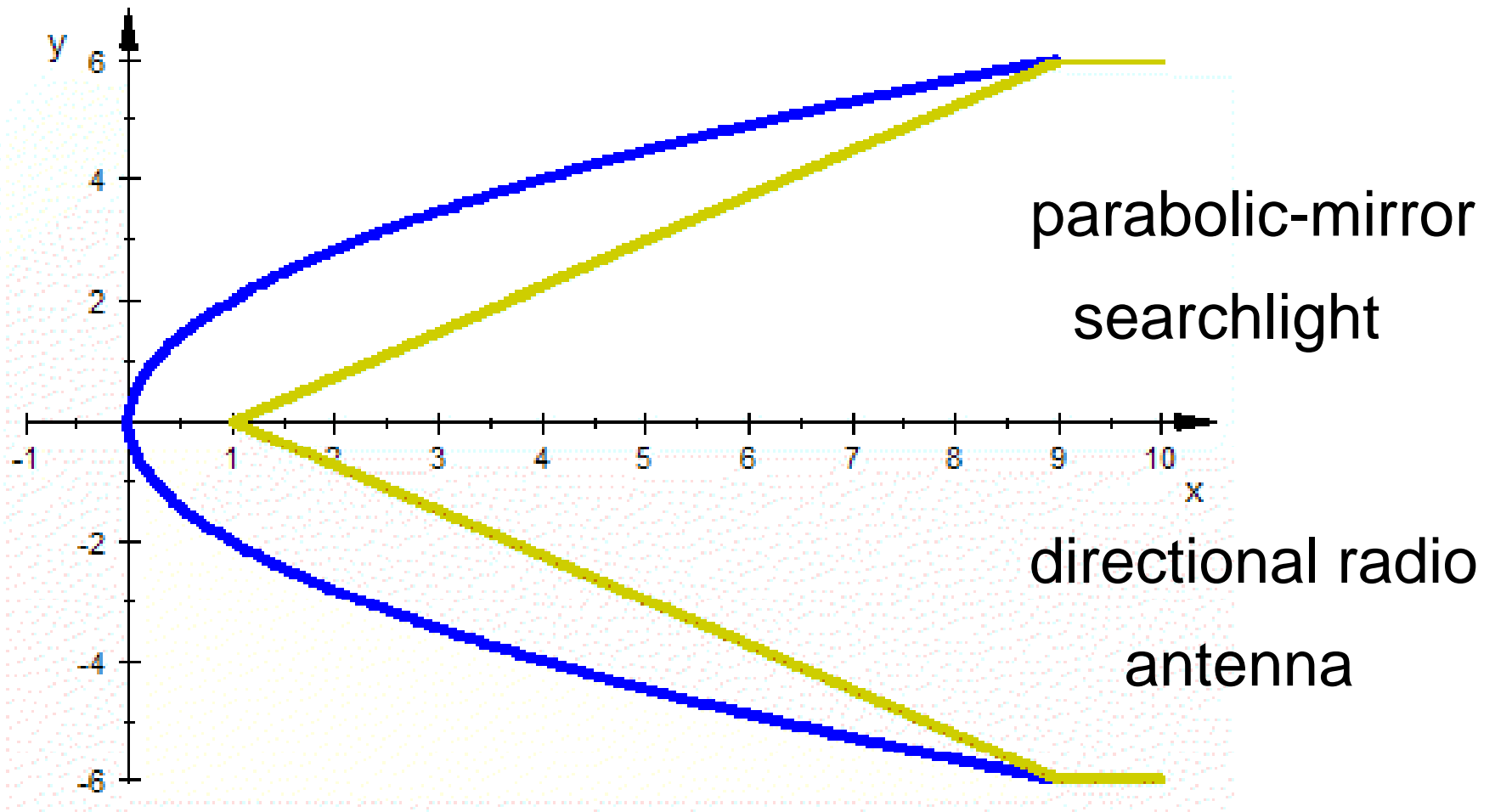


Funktionen als zentrales Werkzeug



Funktionen as a Central Tool



Gliederung: Mathematik für alle

- Moderne Mathematik
 - **Werkzeuge für Mathematik**
- **Funktionen als zentrales Werkzeug**
 - **Phänomene und Strukturen**
- Optimierung als Ziel
 - **Mathematik wird angewendet**
- Numerik findet Lösungen
 - **Mit Hilfe des Computers**

Structure of Mathematics for Everyone

- modern mathematics
 - tool for mathematics
- functions as a central tool
 - Phänomene und Strukturen
- optimization as a goal
 - applying of mathematics
- numerics does solutions find
 - with the help of computers

Mathematik und Sprache

- formale Sprache
 - Mathematiker unter sich, M.-Bücher
- verbale Sprache mit Exaktheitsanspruch
 - Mathematik in anderen Wissenschaften
- offene aber treffende verbale Sprache
 - Ziel von allg. Mathematik-Lehre
- visuell unterstützte Sprache
 - Basis für das Lehren
- Sprache des Lernens und Herantastens₅

Mathematics and Language

- formal language
 - mathematicians each other, m.-books
- verbal language claim for exactness
 - mathematics in other sciences
- open but accurate verbal language
 - goal of general mathematics teaching
- visually aided language
 - basis of learning
- language of approach and first learning

Mathematik und Sprache am Beispiel

Eine Funktion ist stetig im Punkt $B=(a,b)$

- formale Sprache $\forall \varepsilon > 0 \exists \delta > 0 : \forall x \in \mathcal{U}_\delta(a) \Rightarrow f(x) \in \mathcal{U}_\varepsilon(b)$
- verbale Sprache mit Exaktheitsanspruch

Für alle Epsilon größer Null gibt es ein Delta größer Null so, dass für alle x aus einer Delta-Umgebung von a die Funktionswerte in einer Epsilon-Umgebung von b liegen.

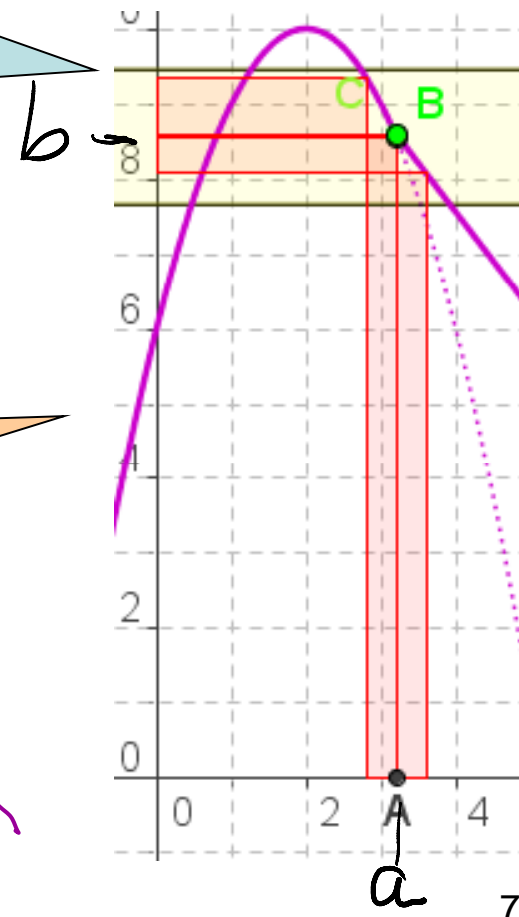
- offene aber treffende verbale Sprache

Wenn die x -Werte von beiden Seiten an a heranrücken, dann rücken die Funktionswerte beliebig dicht an b heran.

- visuell unterstützte Sprache

Man kann dies in einem Zug zeichnen.

- Sprache des Lernens und Herantastens



Mathematics and Language in an Example

A function is continuous at a point $B=(a,b)$

- formal language $\forall \varepsilon > 0 \exists \delta > 0 : \forall x \in \mathcal{U}_\delta(a) \Rightarrow f(x) \in \mathcal{U}_\varepsilon(b)$

- verbal language with claim of exactness

For all epsilon greater than zero there does exist a delta greater zero so that for all x out of a delta neighborhood of the abscissa a the function values lie in an epsilon neighborhood of b.



- open but accurate verbal language

When the x-values approach a by both sides, than the y-values approach arbitrarily close to b.

- visuall aides language

You can skatch then without lifting the pencil.

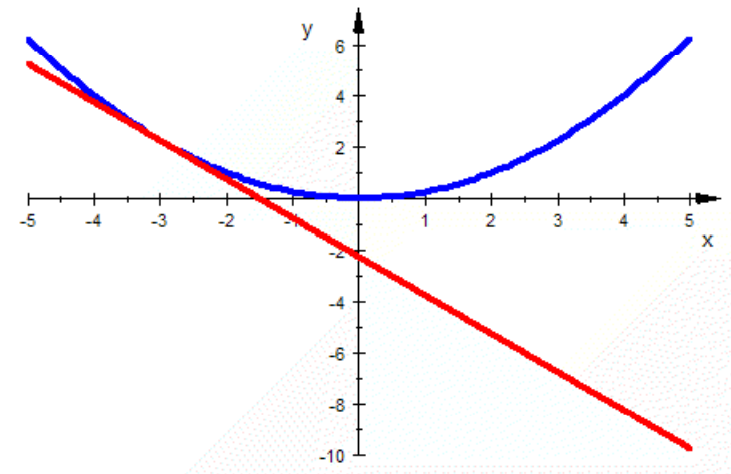
- language of approach and first learning

Aufgabe von „Mathematik für alle“
ist es

Funktionen als zentrales Werkzeug

begreifbar zu machen.

Mit visueller Unterstützung
sollen Sie die Funktionen-
Welt ordnen und gliedern.



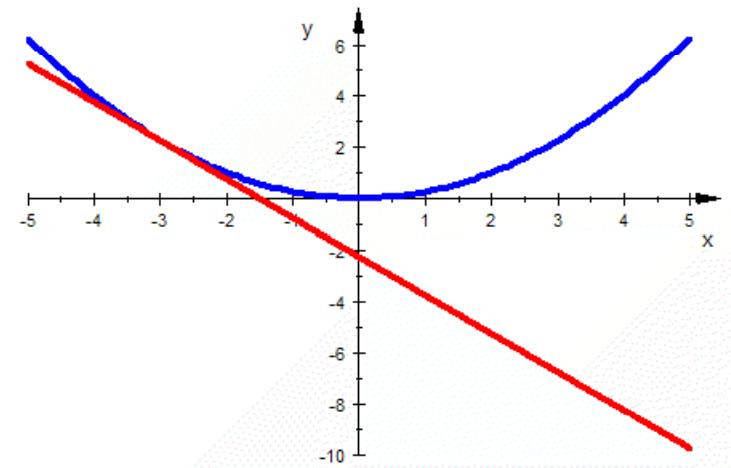
Sie sollen die tragenden Konzepte verstehen und
einen Eindruck vom Nutzen bekommen.

Berechnungen, und Vertiefungen folgen in einigen
Fachrichtungen später. Aber nicht hier!!!!!!

The Challenge of „Mathematik for Everyone“
is to support you in grasping:

functions are a central tool

You shall put in order and
classify the world of functions
by visual support.



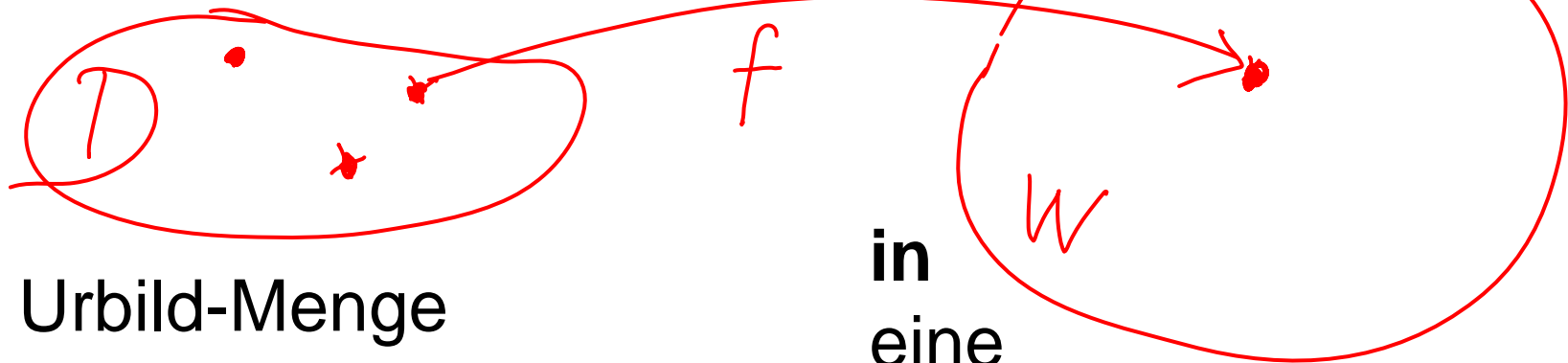
You shall grasp the leading concepts and shall get an
impression of the convenience of mathematics.

Calculations and consolidations will follow later in some
branches of study. But not here!!!!!!

Was ist überhaupt eine Funktion?

Abbildung, Funktion und Zuordnung sind Synonyme.

Es wird eine Definitionsmenge



Urbild-Menge

$$f: D \longrightarrow W$$

Wertemenge abgebildet
Bildmenge

und zwar auf **eindeutige** Weise.

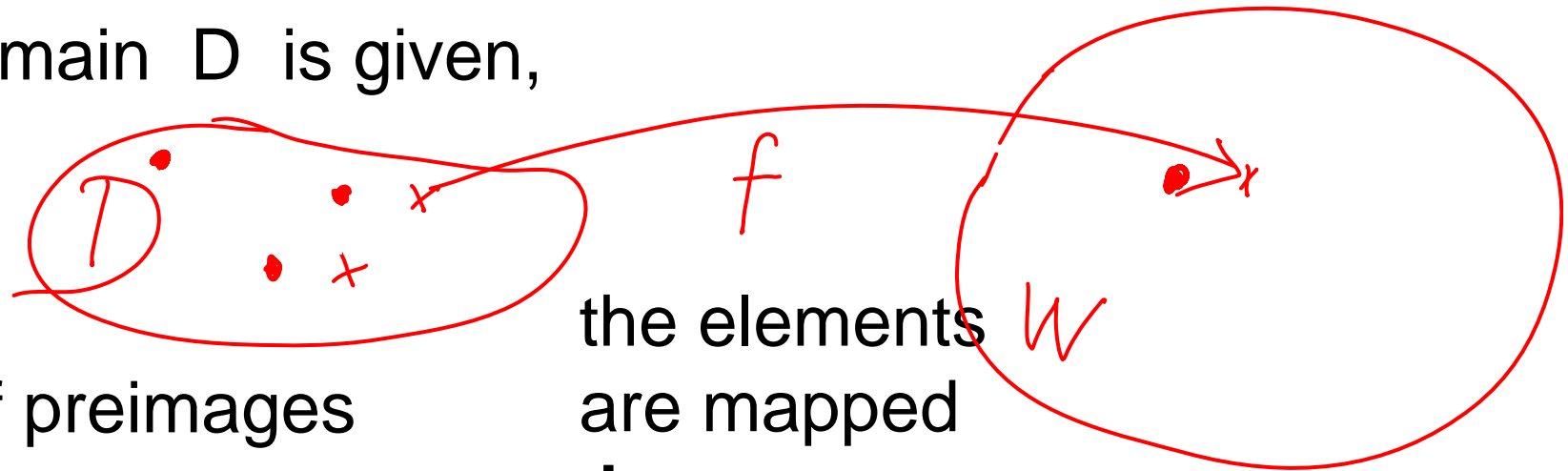
d.h. jedes Urbild hat ein Bild, aber auch nur eins.

d.h. jedes Urbild hat genau ein Bild.

What's a Function Generally?

mapping, function and assigning are synonyms.

A domain D is given,



set of preimages

the elements
are mapped

in

a range of values W

$$f: D \longrightarrow W$$

in a **unique** matter.

d.h. every preimage has an images, but only one image.

d.h. every preimage has exactly one image.

Ausschärfung der Begriffe

Abbildung, Funktion und Zuordnung sind Synonyme.

fast

Abbildung verwendet man allgemein, im Besonderen aber in der Geometrie:

Spiegelung, Drehung, Scherung, Projektion....

Zuordnung nimmt den Vorgang des Zuordnens und die einzelnen Objekte stärker in den Blick: den Waren sind Preise zugeordnet, jedem Konto eine PIN,...

Schule bis Klasse 8

Funktion nimmt die Veränderung stärker in den Blick: z.B. der Druck ist eine Funktion der Temperatur.
„y ist eine Funktion von x“

Spezifikation of the Concepts

mapping, function and assigning are synonyms.

almost

mapping is taken generally, but specially in geometry:

reflection, rotation, shearing, projection....

assigning looks at the process: an object could be assigned to another:

every article is assigned to its price, every account is assigned to a PIN,... This concept is common in school.

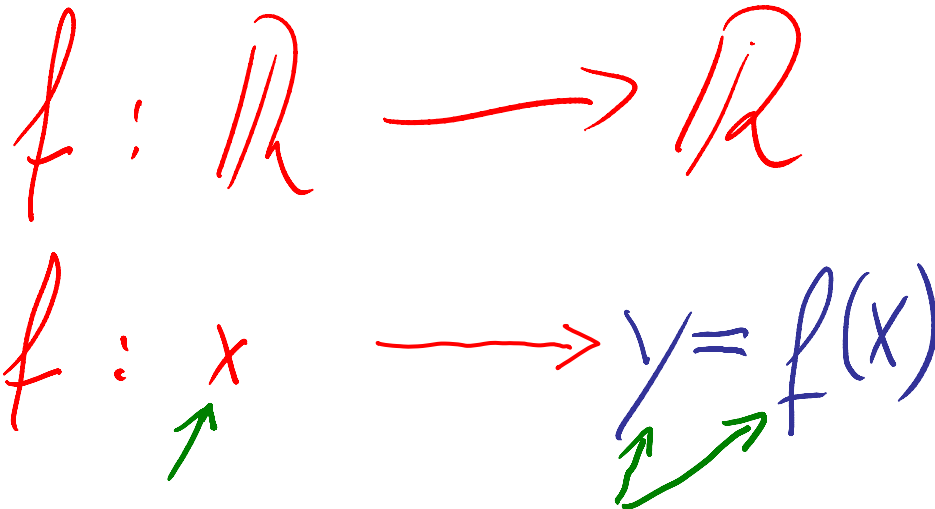
function looks at the transformation:

i.e. the pressure is a function of temperature.

„y is a function of x“

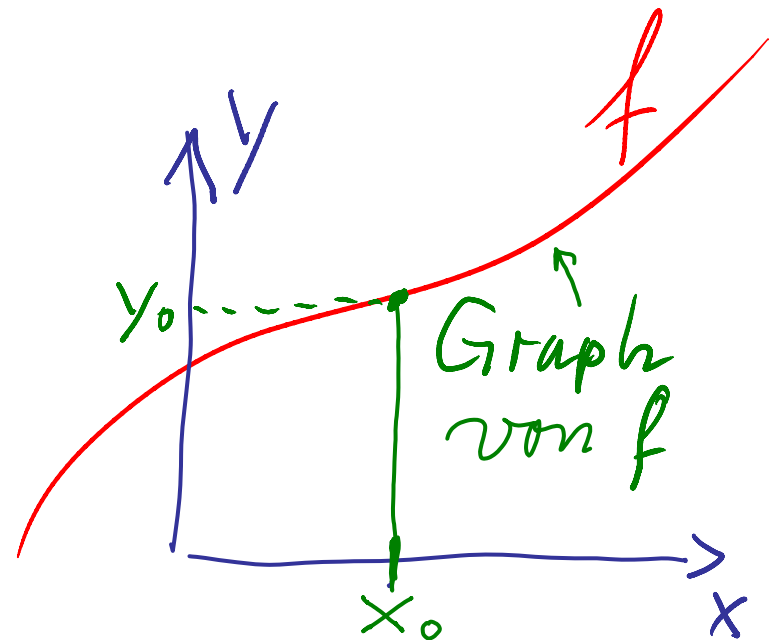
„y ist eine Funktion von x“

Wir betrachten nun erstmal den wichtigen Spezialfall, bei dem die reellen Zahlen in sich abgebildet werden.



Stelle,
Abszisse,
x-Wert,
Einsetzung,
Argument
unabh. Variable

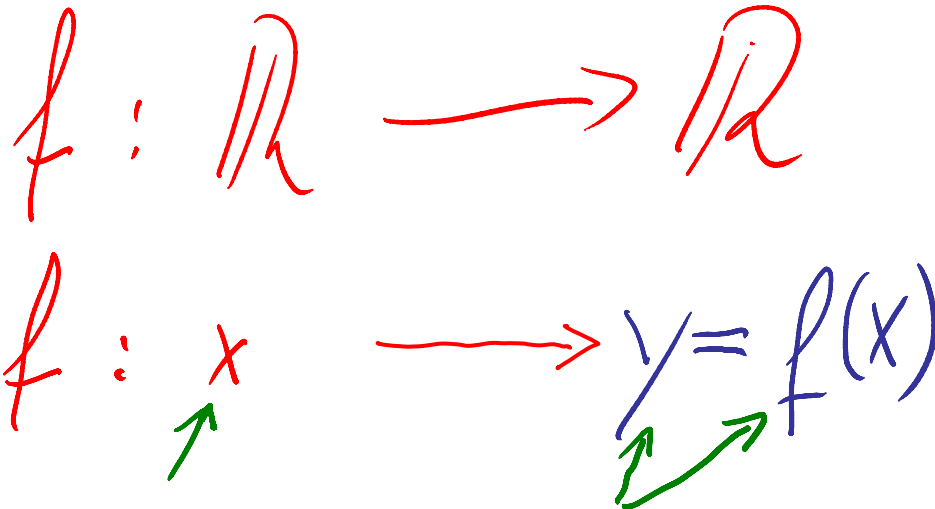
Wert
Ordinate
y-Wert,
Funktionswert
abhängige Variable



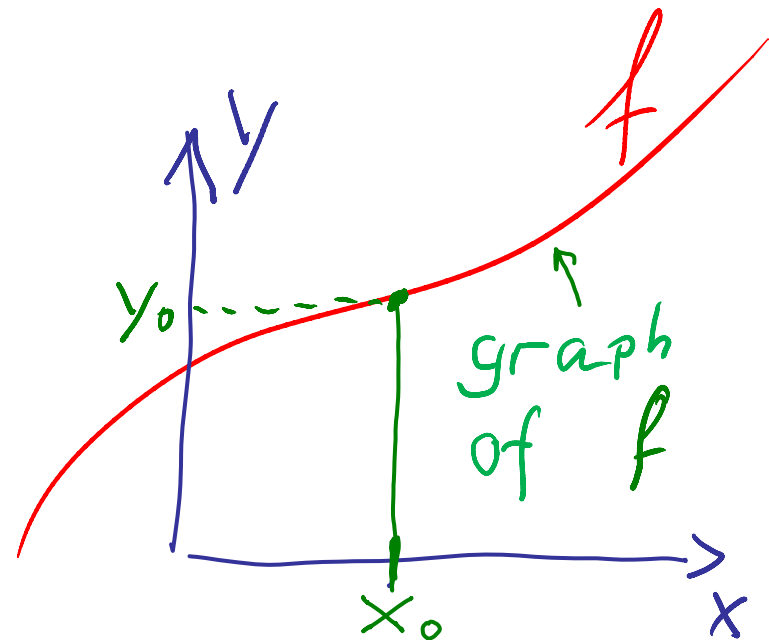
Die Funktion heißt **f**

„y is a Function of x“

At first we are looking at a special situation:
real numbers will be mapped in real numbers.

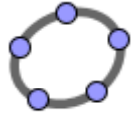


place,	value
abscissa,	ordinate
x-value,	y-value,
substitution,	function value
argument	dependent variable
independent variable	



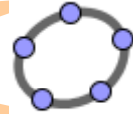
The function is named **f**.

Funktionsgleichung $f(x) = x^k$



Potenzfunktion

Grundtyp Potenzfunktion



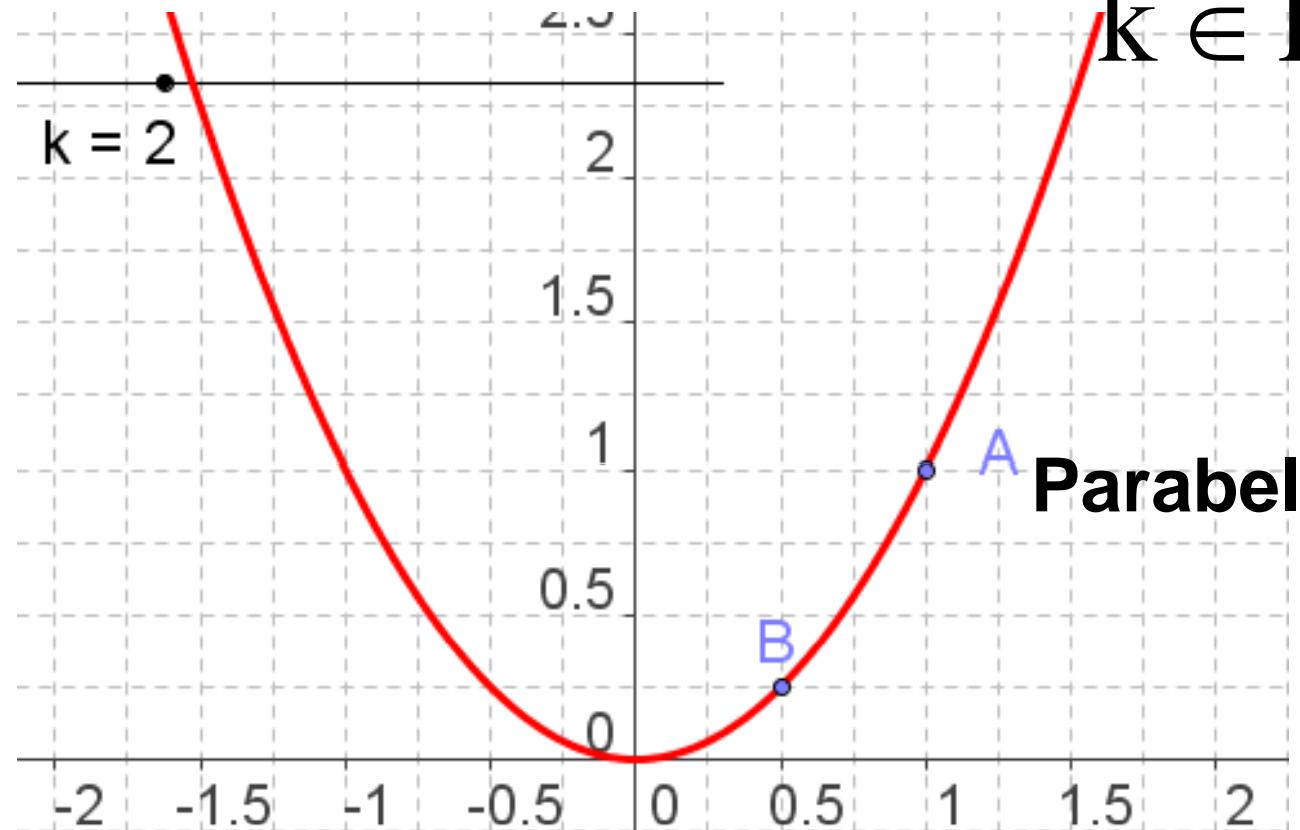
Potenzfunktion enger

$k \in \mathbb{R}$

Hauptform:

$$f(x) = x^2$$

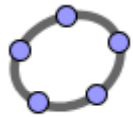
$k \in \mathbb{R}$



GeoGebra, freies Mathematikwerkzeug, www.geogebra.org

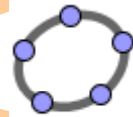
17

Equation of a Function $f(x) = x^k$



Potenzfunktion

Basic type power function



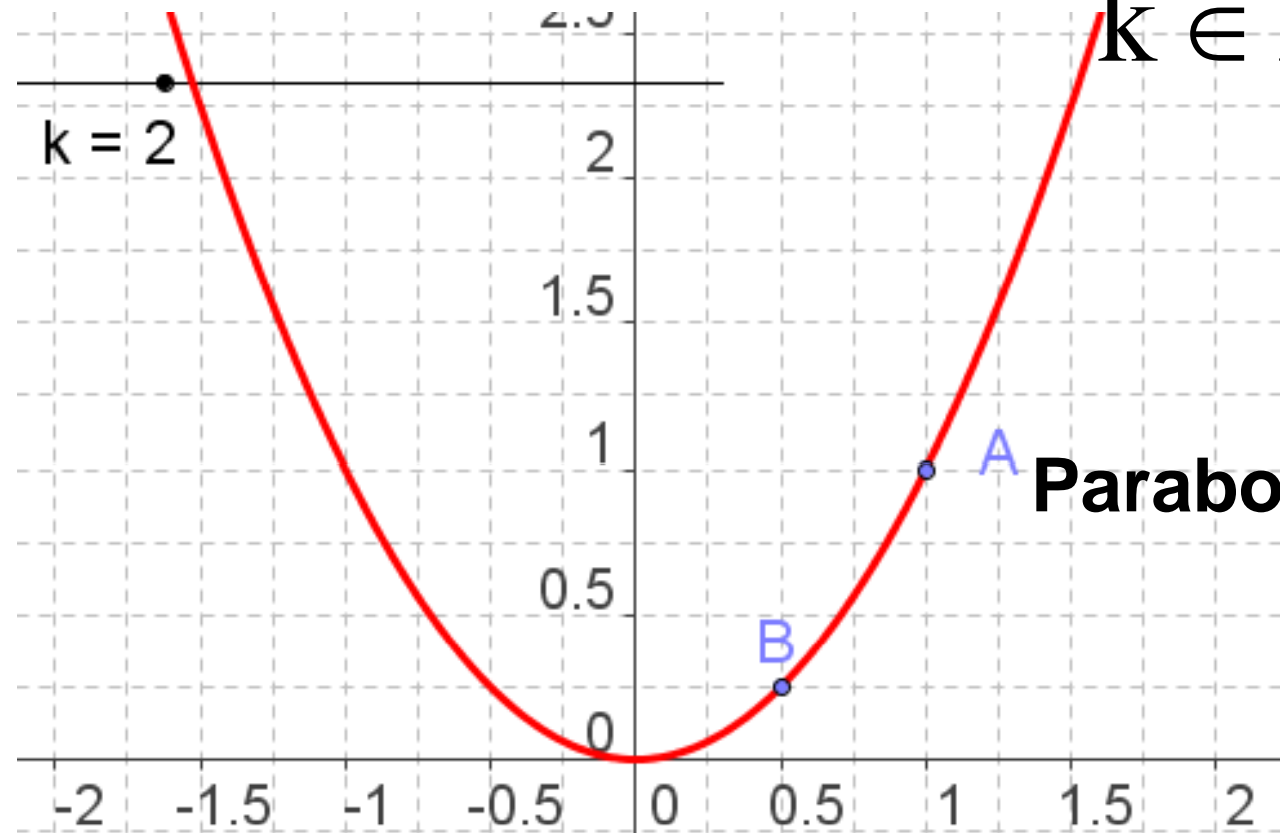
Potenzfunktion enger

$k \in \mathbb{R}$

main form:

$$f(x) = x^2$$

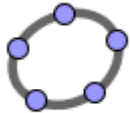
$k \in \mathbb{R}$



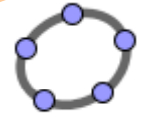
Parabola

GeoGebra, free tool for mathematics www.geogebra.org

Funktionsgleichung $f(x) = x^k$



Grundtyp Potenzfunktion

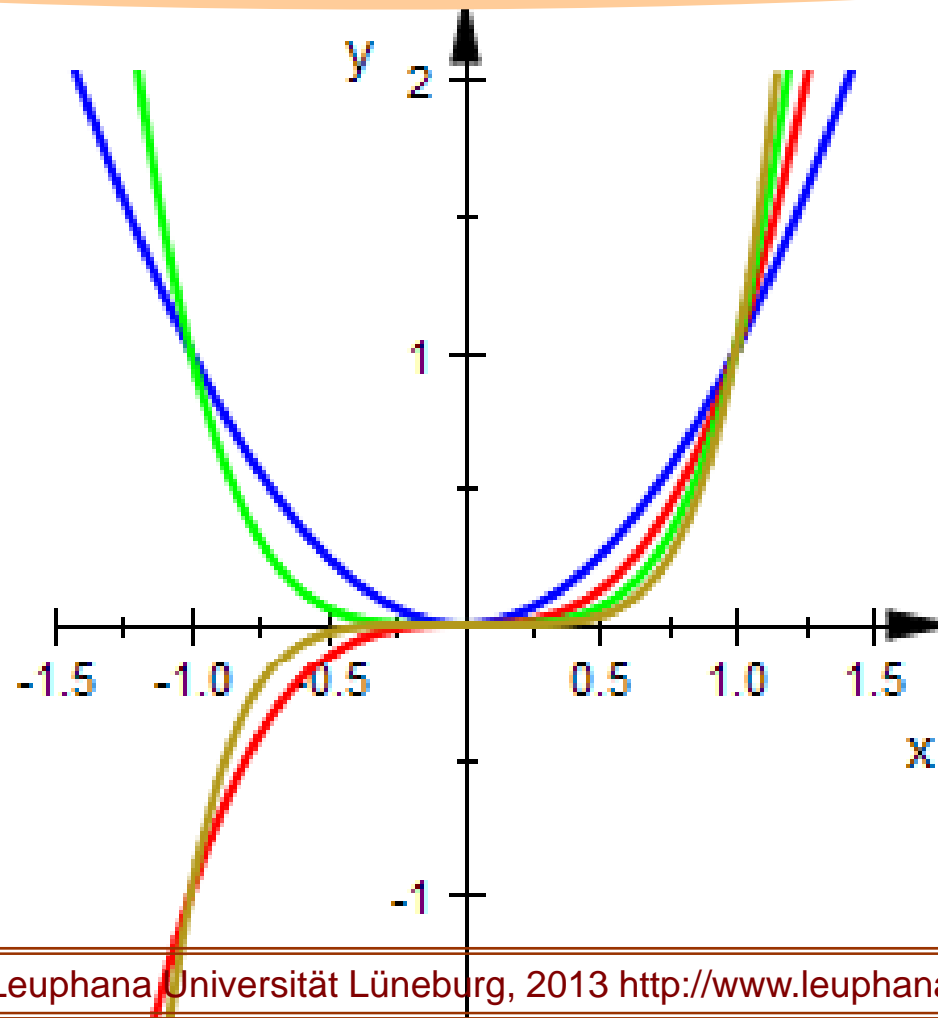


Potenzfunktion

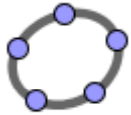
Hauptform:

$$k > 1$$

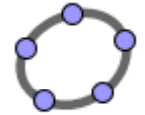
Potenzfunktion enger



Equation of the Function $f(x) = x^k$



basic type power function

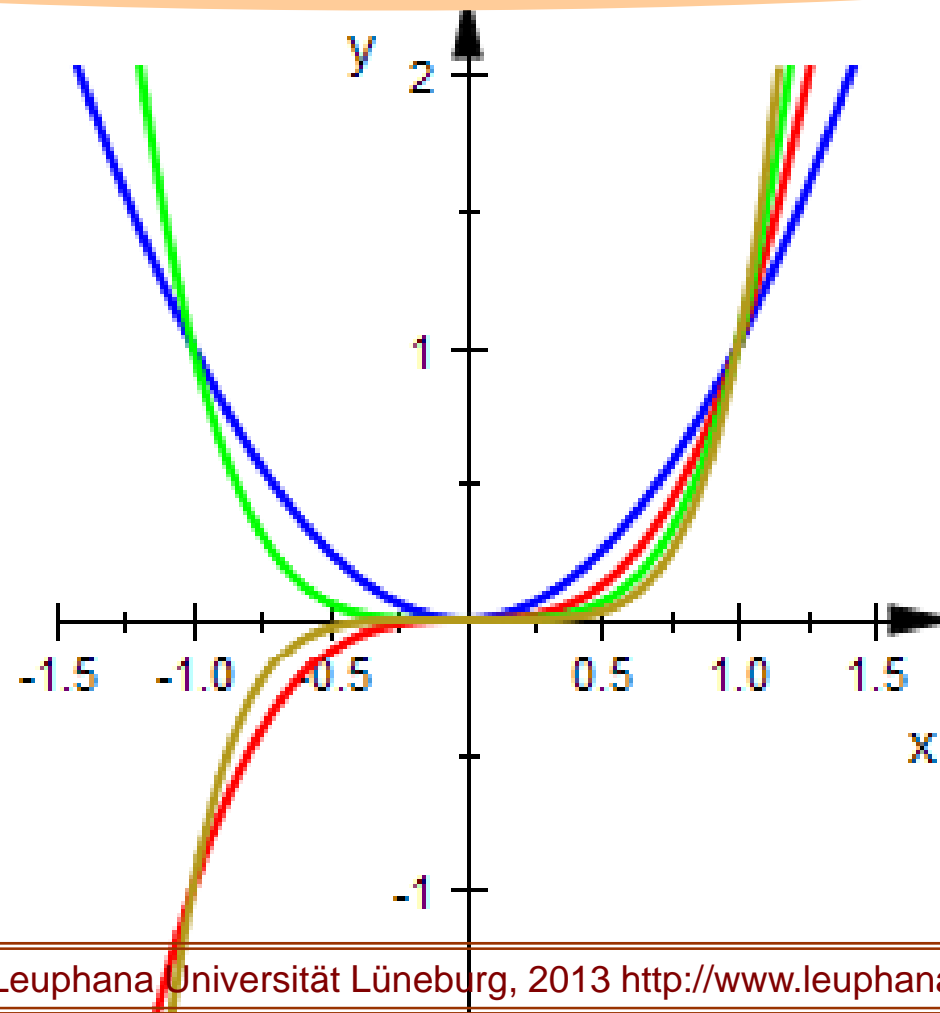


Potenzfunktion

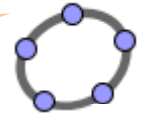
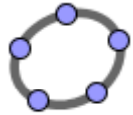
main form:

$$k > 1$$

Potenzfunktion enger



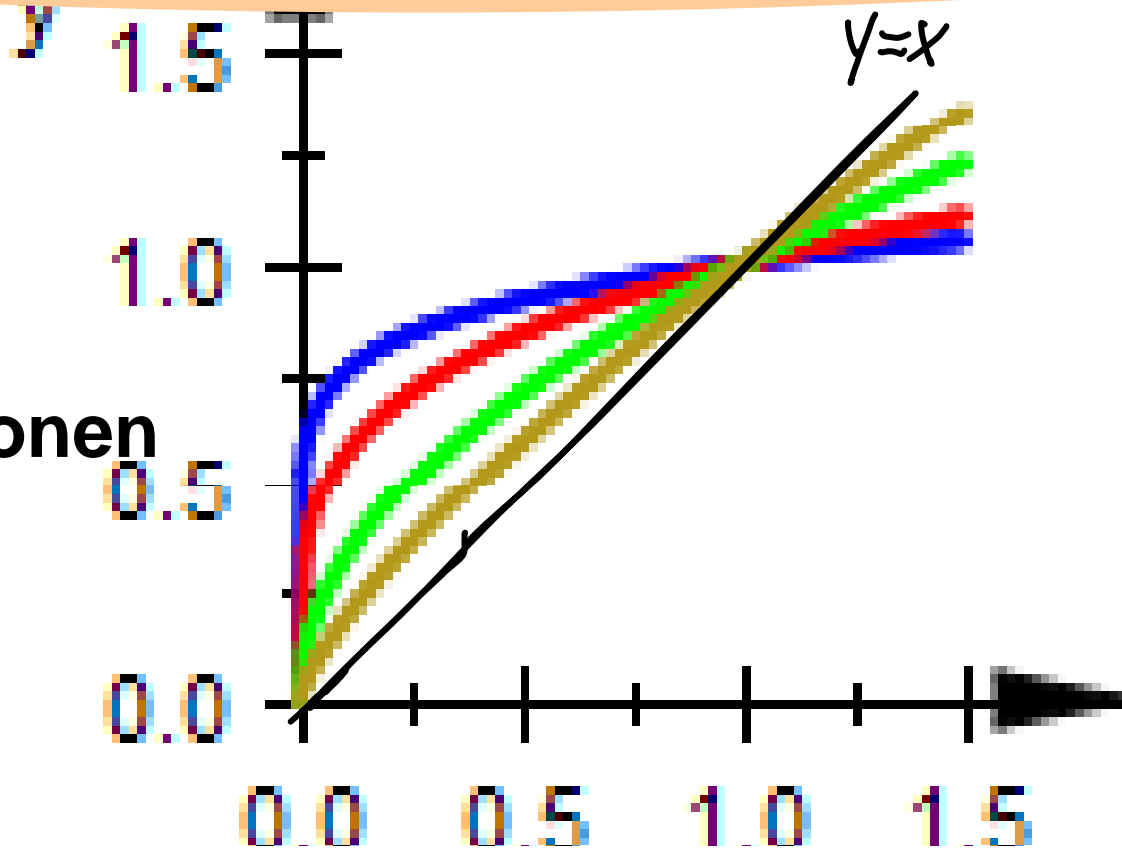
Funktionsgleichung $f(x) = x^k$



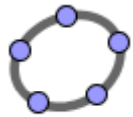
Grundtyp Potenzfunktion

$$0 < k < 1$$

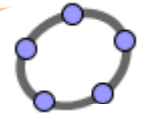
Wurzelfunktionen



Equation of the Function $f(x) = x^k$

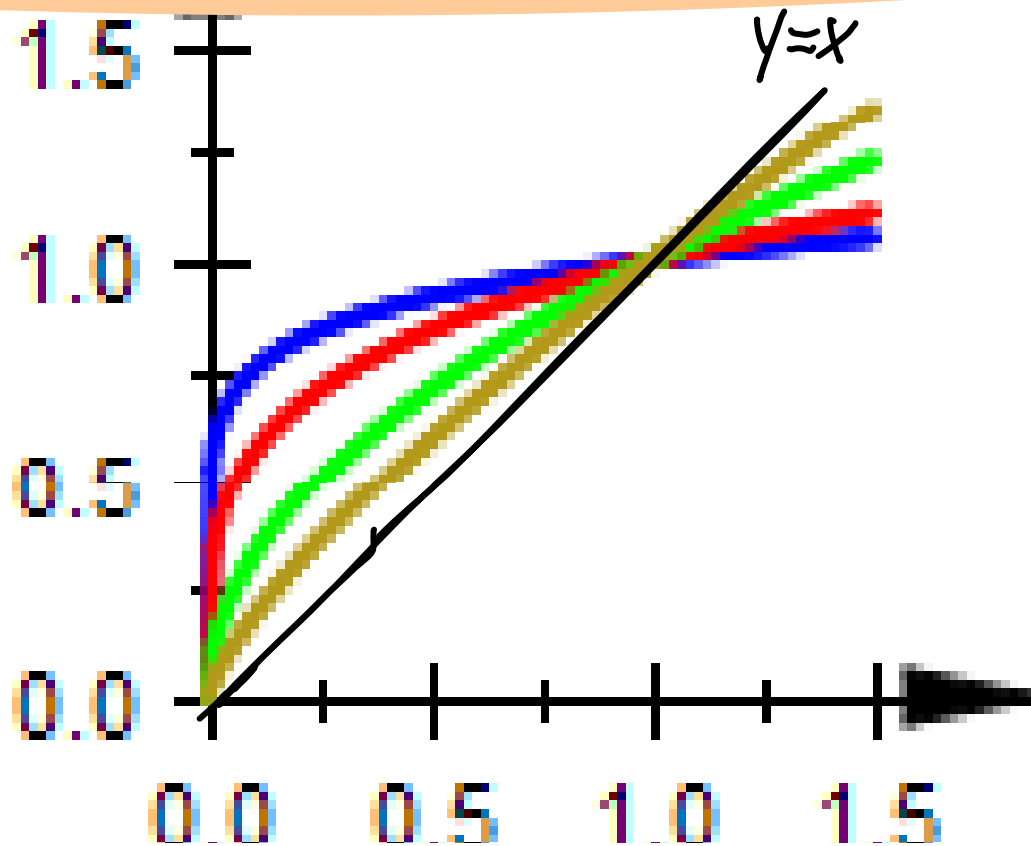


basic type power function

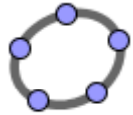


$$0 < k < 1$$

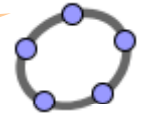
root functions



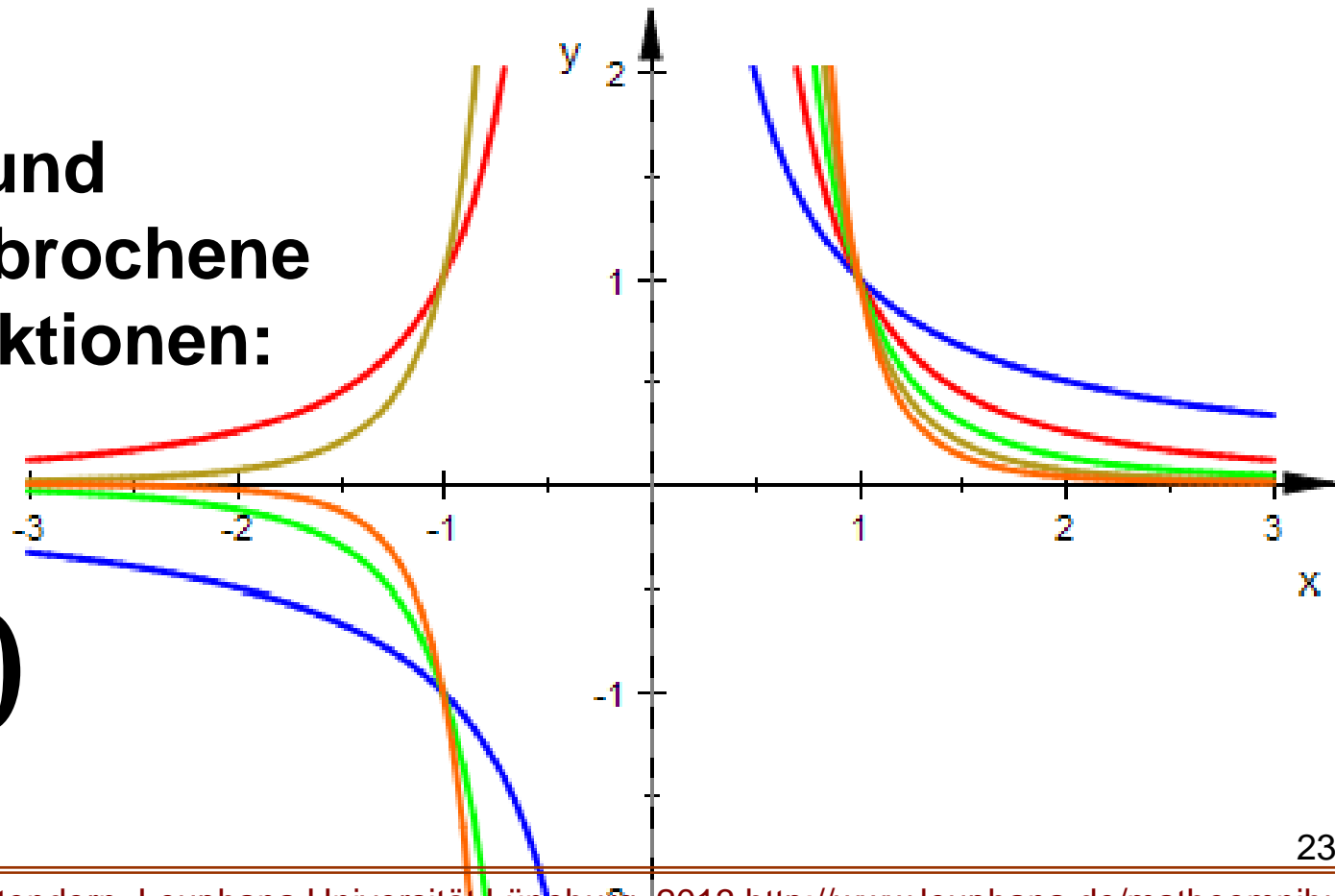
Funktionsgleichung $f(x) = x^k$



Grundtyp Potenzfunktion

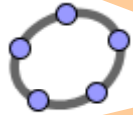


**Hyperbel und
andere gebrochene
Potenzfunktionen:**

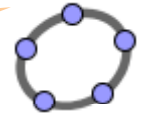


$$k < 0$$

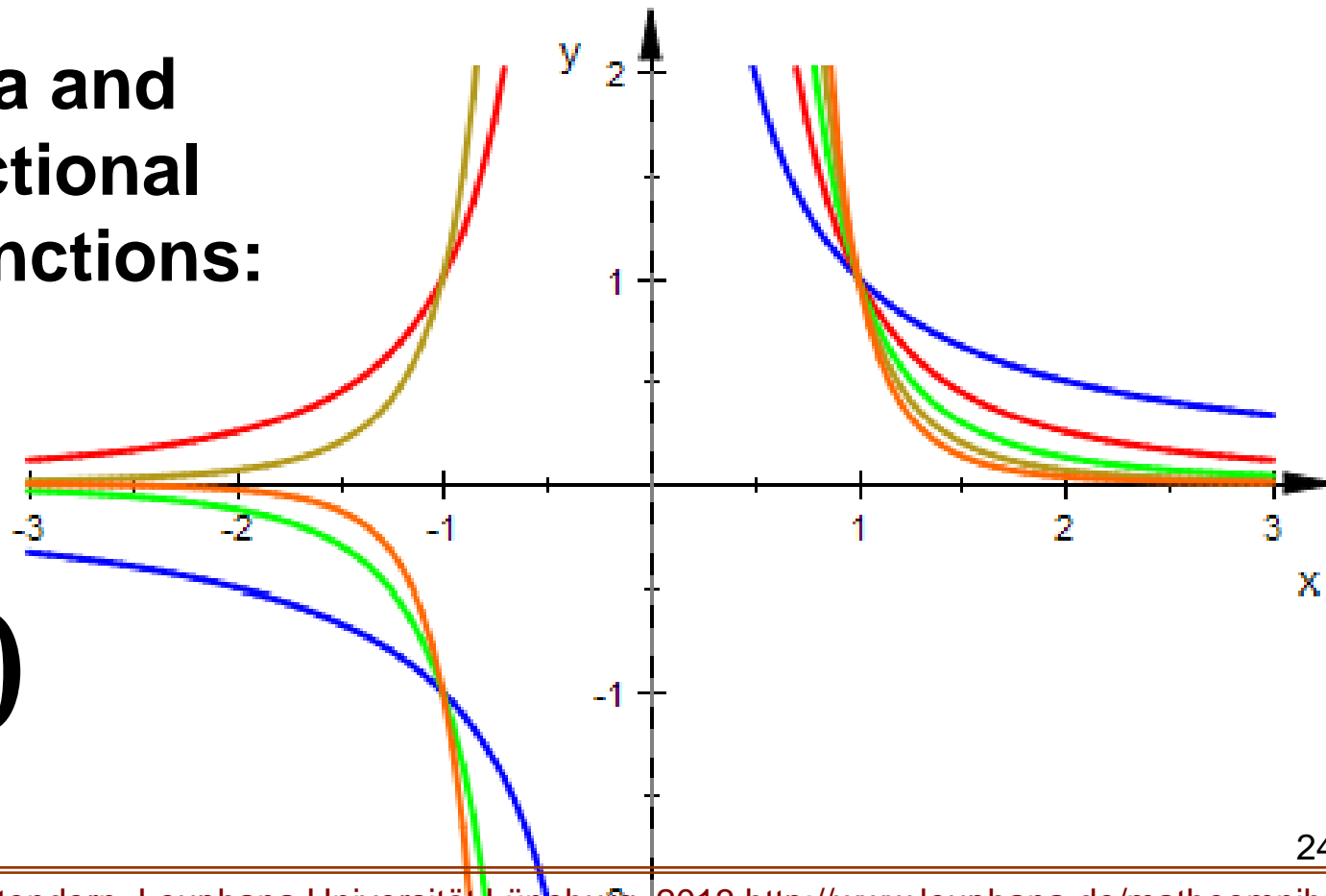
Equation of the Function $f(x) = x^k$



basic type power function



hyperbola and
other fractional
power functions:

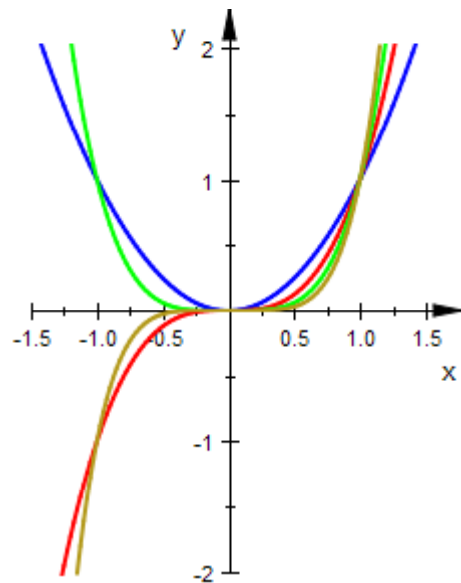


$k < 0$

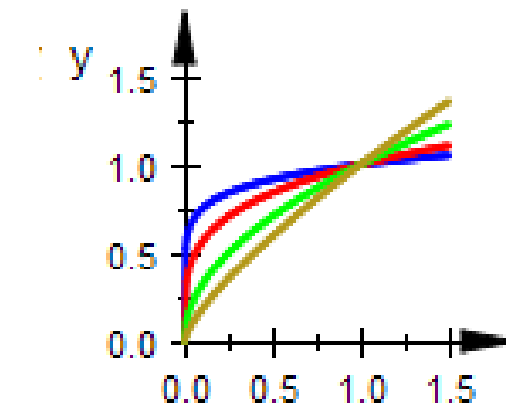
Funktionsgleichung $f(x) = x^k$

Grundtyp Potenzfunktion

Hauptform:

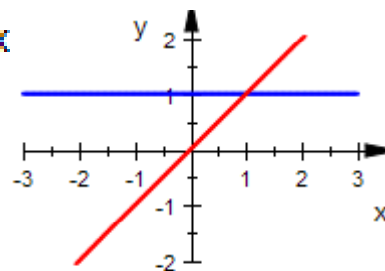


$$k \in \mathbb{N}$$

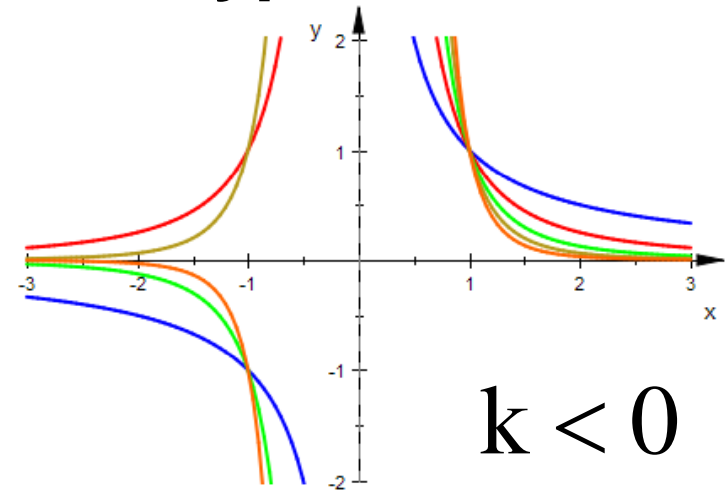


$$0 < k < 1$$

$$k = 0; \quad k = 1$$



Hyperbel u.a.

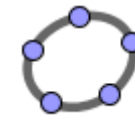
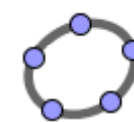


$$k < 0$$

GeoGebra

Potenzfunktion

Potenzfunktion enger

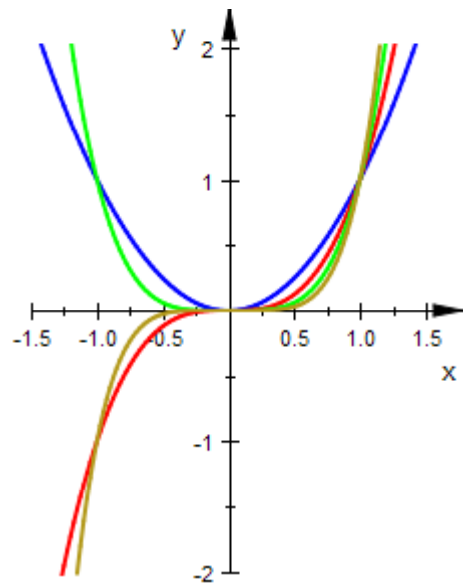


20

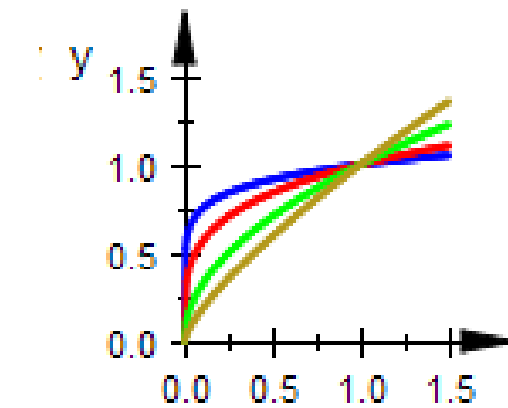
Equation of the Function $f(x) = x^k$

basic type power function

main form:

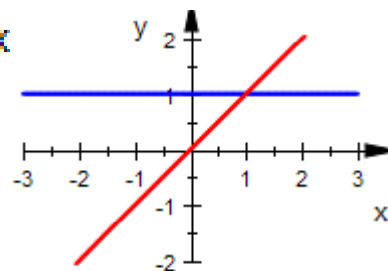


$k \in \mathbb{N}$

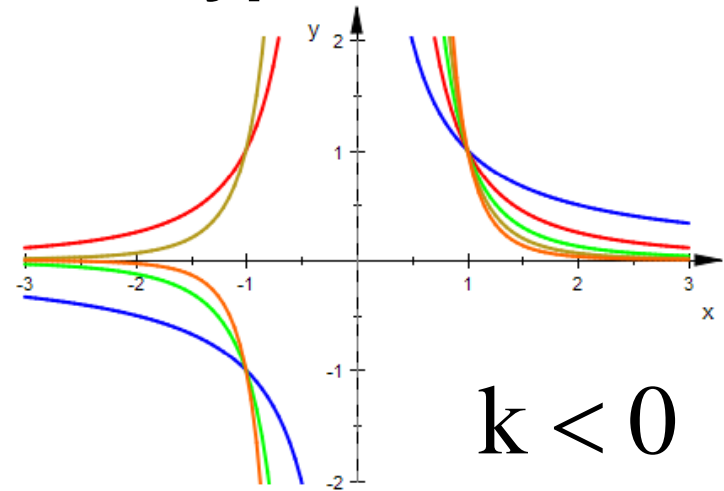


$0 < k < 1$

$k = 0; k = 1$



hyperbolical a.o.

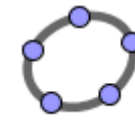
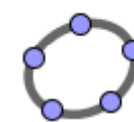


$k < 0$

GeoGebra

Potenzfunktion

Potenzfunktion enger

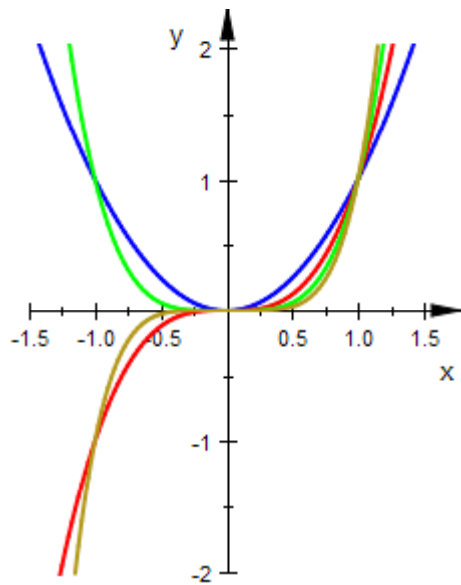


Z0

Funktionsgleichung $f(x) = x^k$

Grundtyp Potenzfunktion

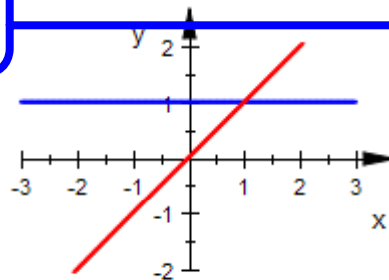
Hauptform: $f(x) = x^k$



$k \in \mathbb{N}$

Grundbausteine für Polynome

Alle GeoGebra-Dateien
findet man in
matheomnibus

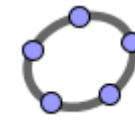
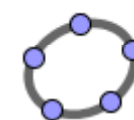


$k = 0; k = 1$

GeoGebra

Potenzfunktion

Potenzfunktion enger

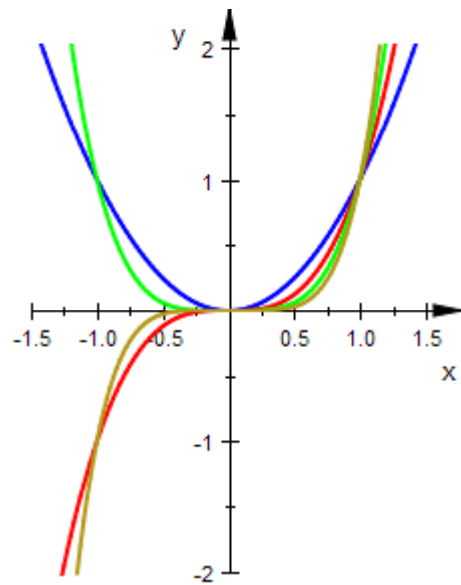


LI

Equation of the Function $f(x) = x^k$

basic type power function

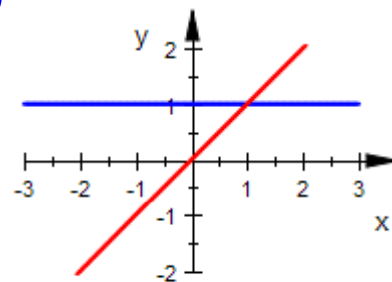
main form: $f(x) = x^k$



$k \in \mathbb{N}$

basic blocks for polynomials

You will find all GeoGebra-files in matheomnibus

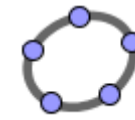
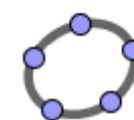


$k = 0; k = 1$

GeoGebra

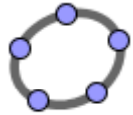
Potenzfunktion

Potenzfunktion enger

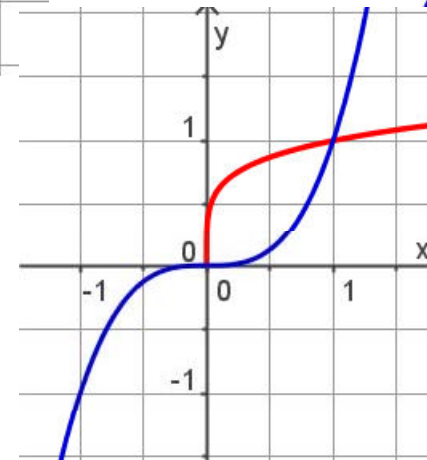
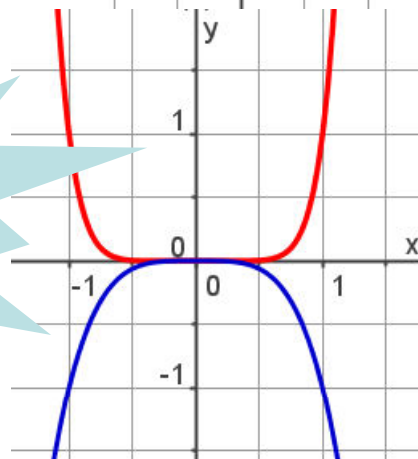
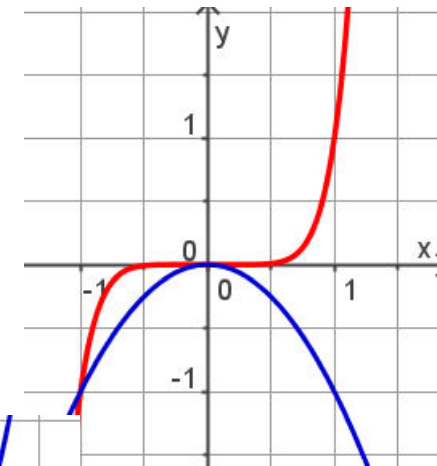
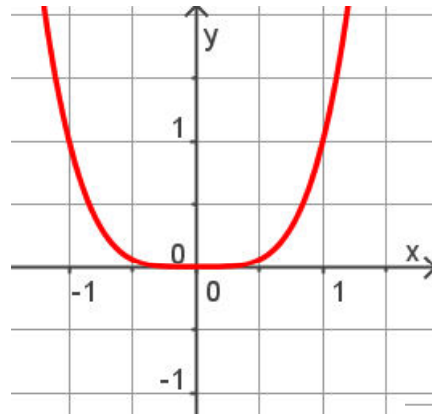
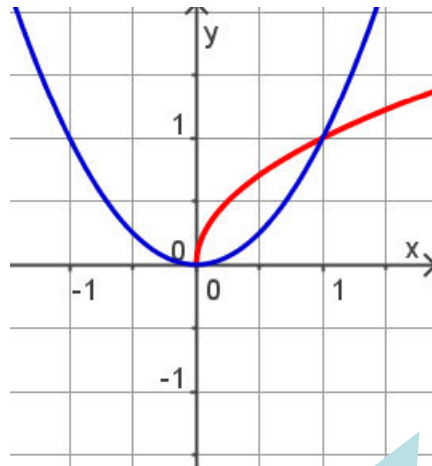
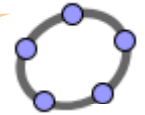


20

Funktionsgleichung $f(x) = t \cdot x^k$



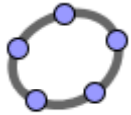
Grundtyp Potenzfunktion



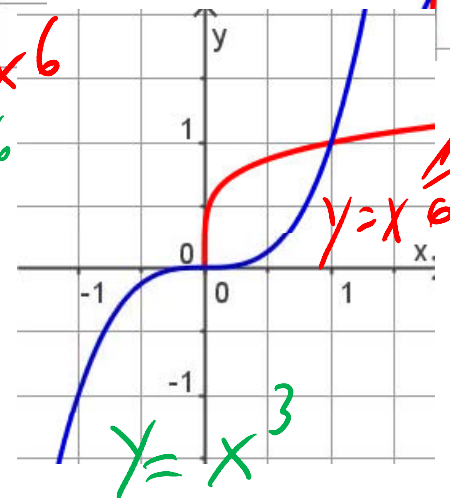
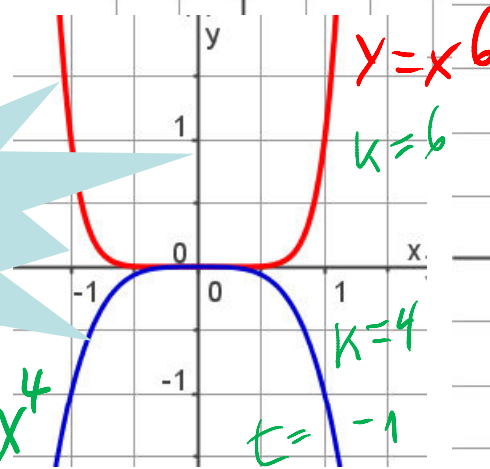
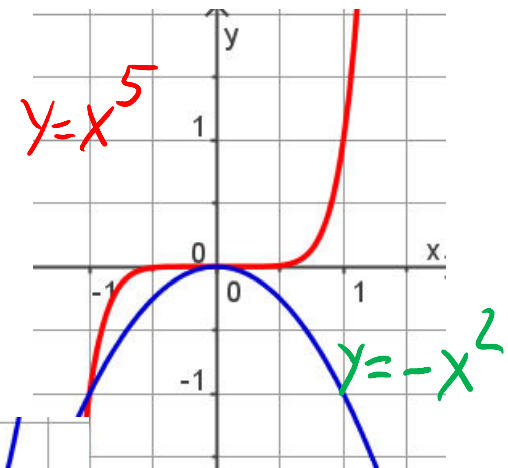
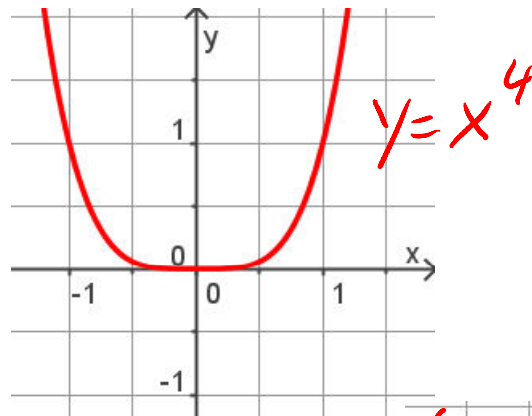
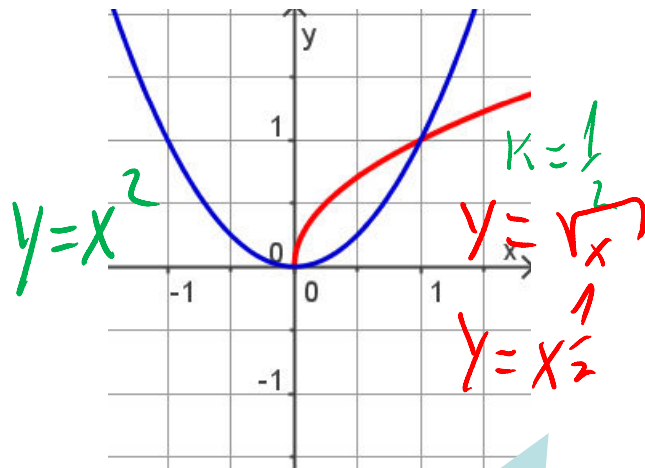
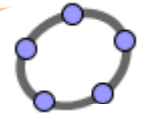
**Selber
machen**

$t = 1$
oder
 $t = -1$

Equation of the Function $f(x) = t \cdot x^k$



basic type power function



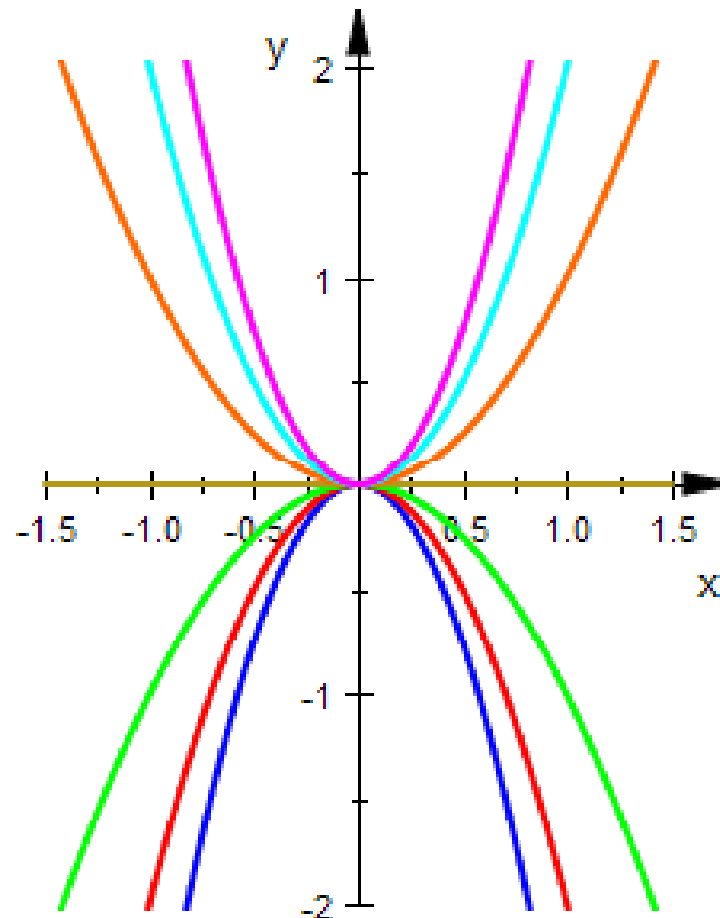
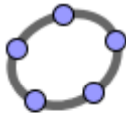
$t = 1$
oder
 $t = -1$

do it yourself

Funktionsgleichung $f(x) = t \cdot x^k$

Variationen in Lage und Form

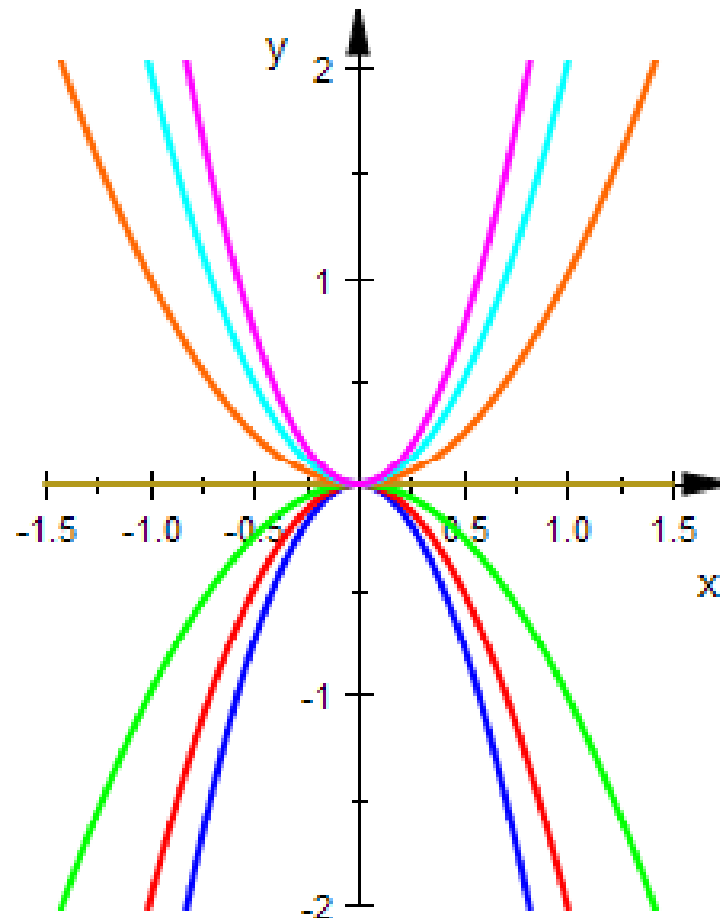
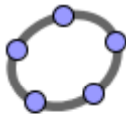
**Strecken, Stauchen,
Spiegeln**



Equation of the Function $f(x) = t \cdot x^k$

variations of location and form

stretching,
compressing,
reflecting



Funktions-Variation

$$f(x) = t(x - a)^k + b$$

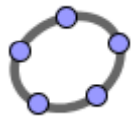
Variationen in Lage und Form

Scheitel

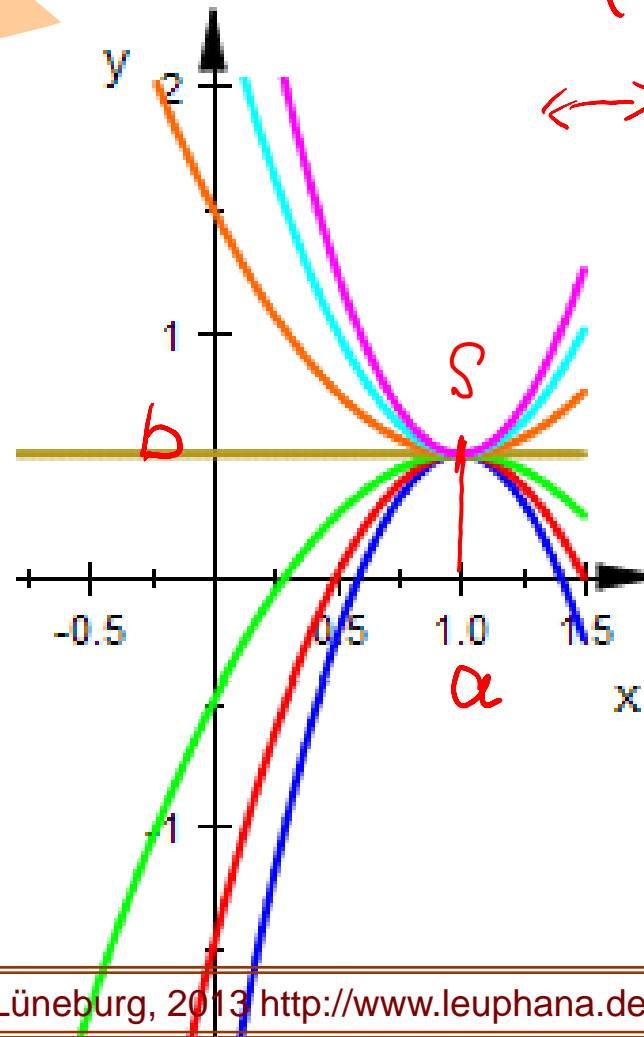
(a/b)



Der Scheitel S ist auf (a,b) verschoben



verschieben

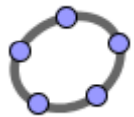


Function Variation

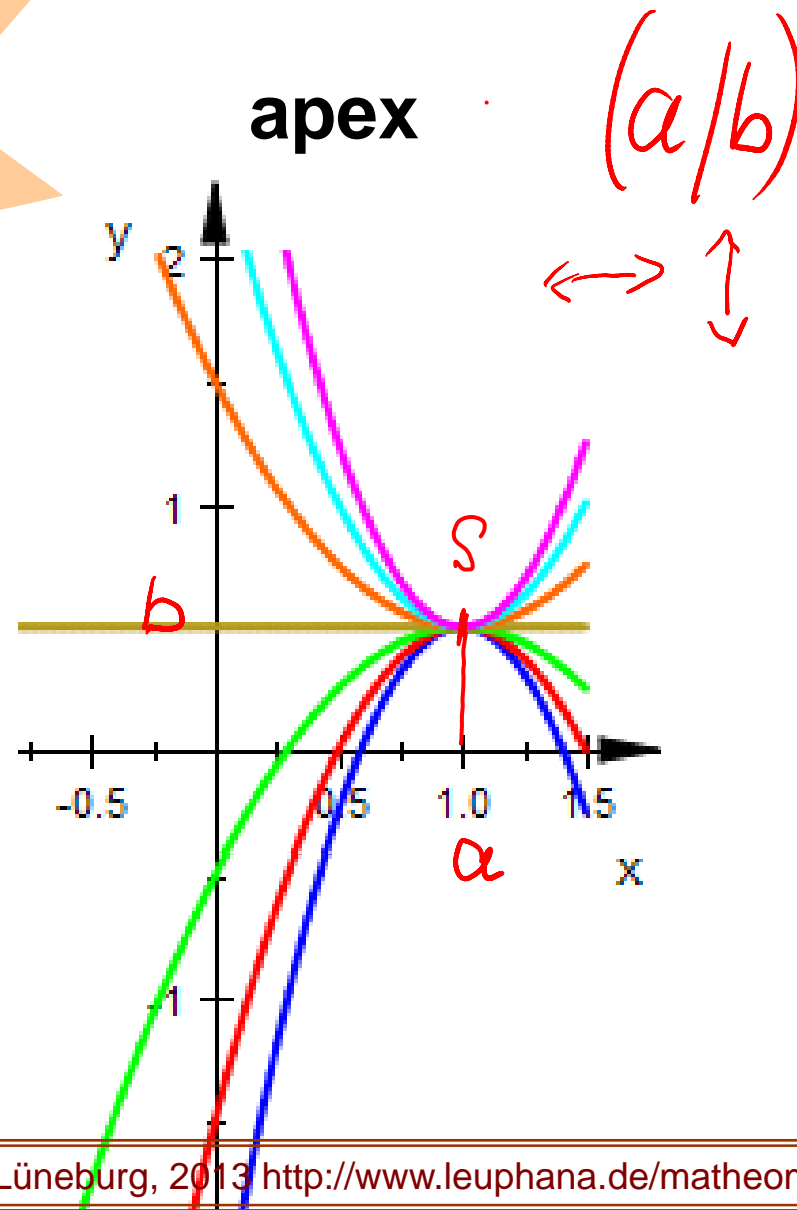
$$f(x) = t(x - a)^k + b$$

variations of
location and form

the apex **S** is scrolled
to point **(a,b)**

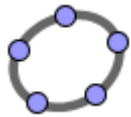


scrolling



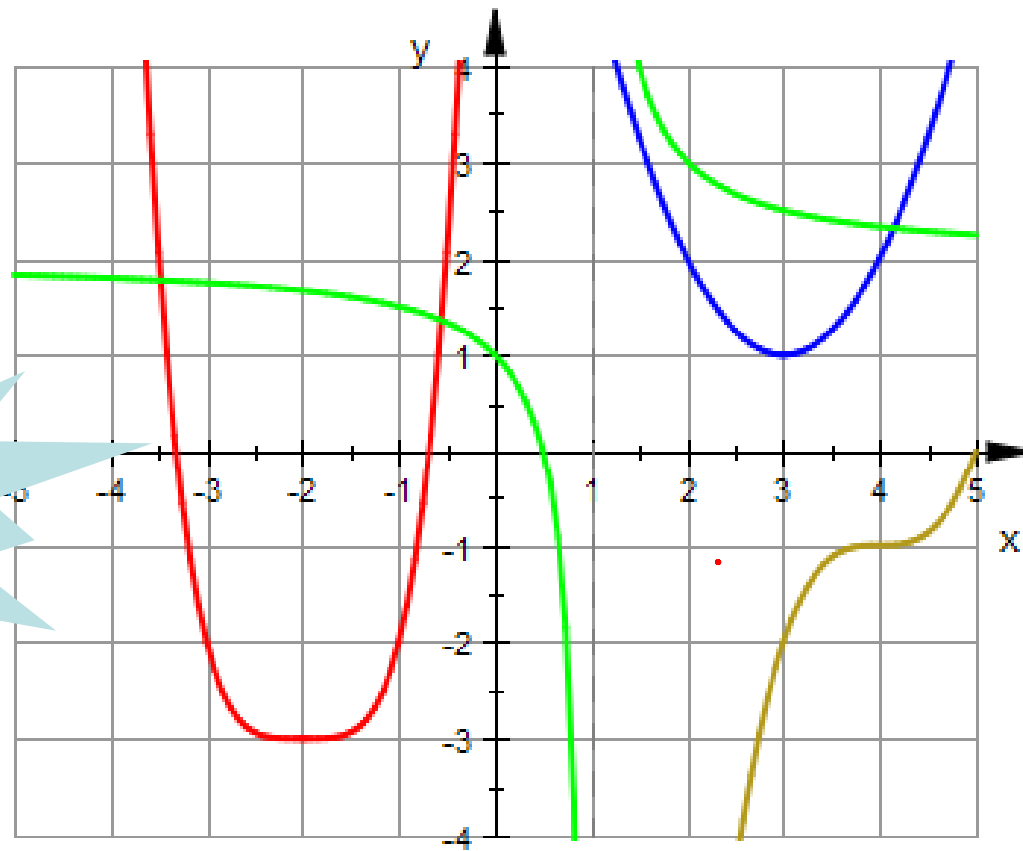
Funktionsgleichung $f(x) = \pm(x - a)^k + b$

Übung mit Potenzfunktionen



verschieben

**Selber
machen**

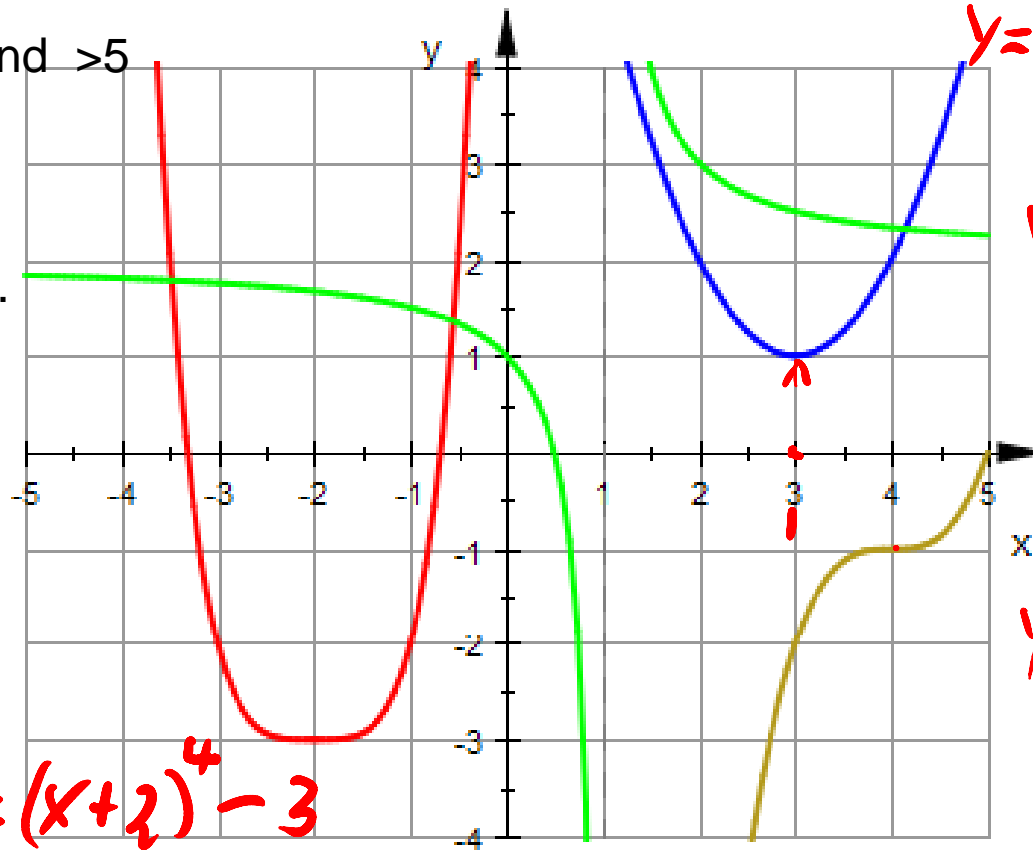


Equation of the Funktion

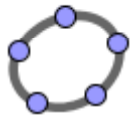
$$f(x) = \pm(x - a)^k + b$$

exercises with power functions

You can not identify
the exponents >4 and >5
Accepted are 4, 6
and 8 too,
respectively
5 und 7 and so on.



verschieben

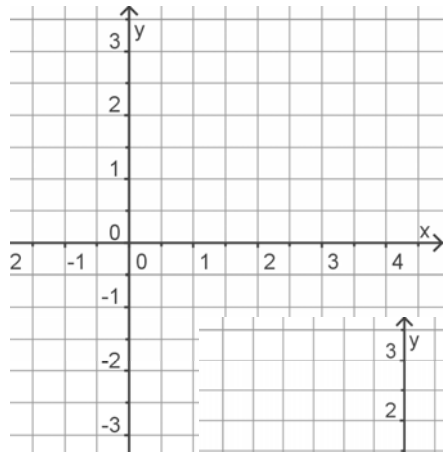


Funktionsgleichung $f(x) = \pm(x - a)^k + b$

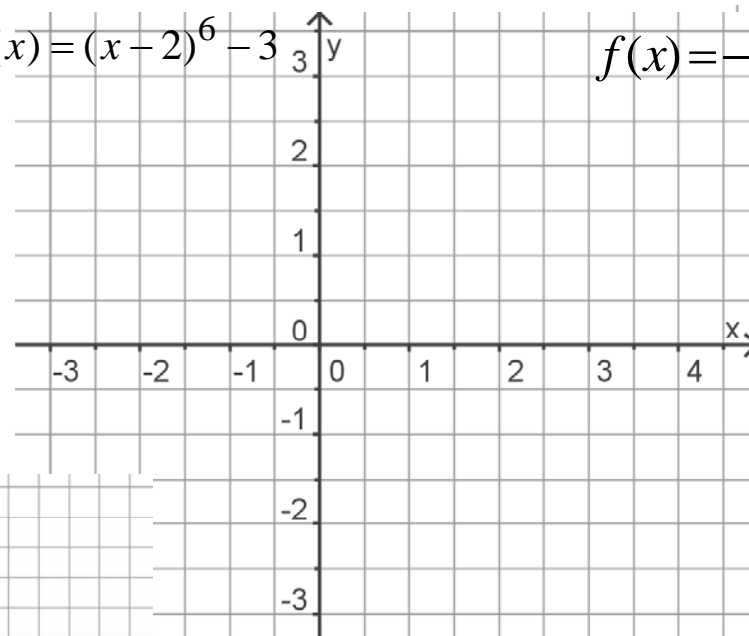
Selber
machen

Übung mit Potenzfunktionen

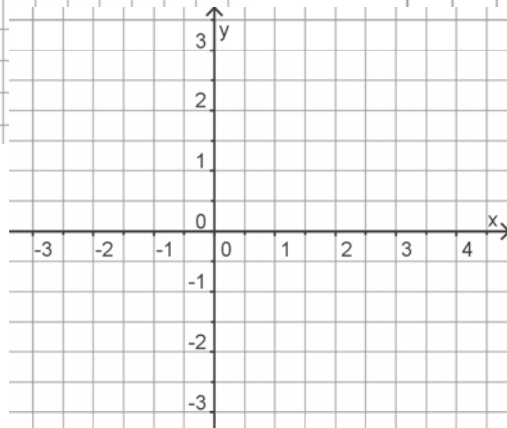
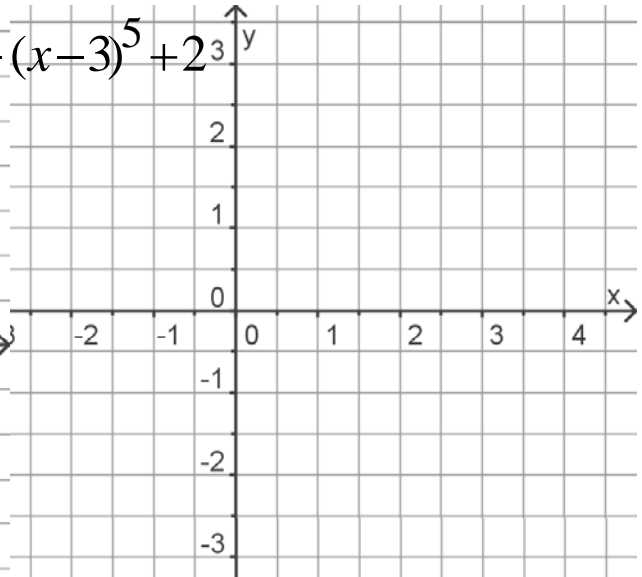
$$f(x) = (x - 3)^4 + 1$$



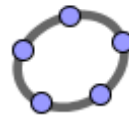
$$f(x) = (x - 2)^6 - 3$$



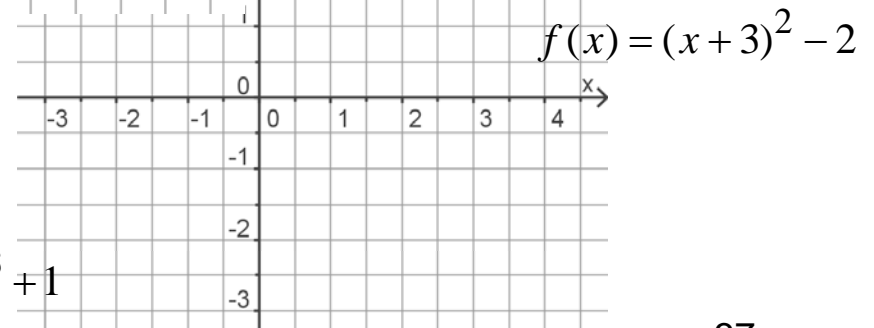
$$f(x) = -(x - 3)^5 + 2$$



$$f(x) = -(x - 3)^4 - 1$$



$$f(x) = -(x + 2)^3 + 1$$

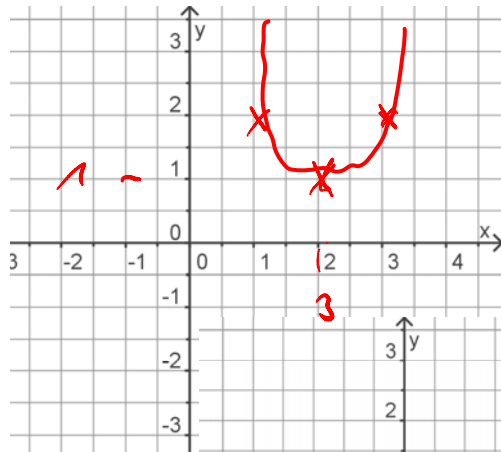


Equation of the Funktion $f(x) = \pm(x - a)^k + b$

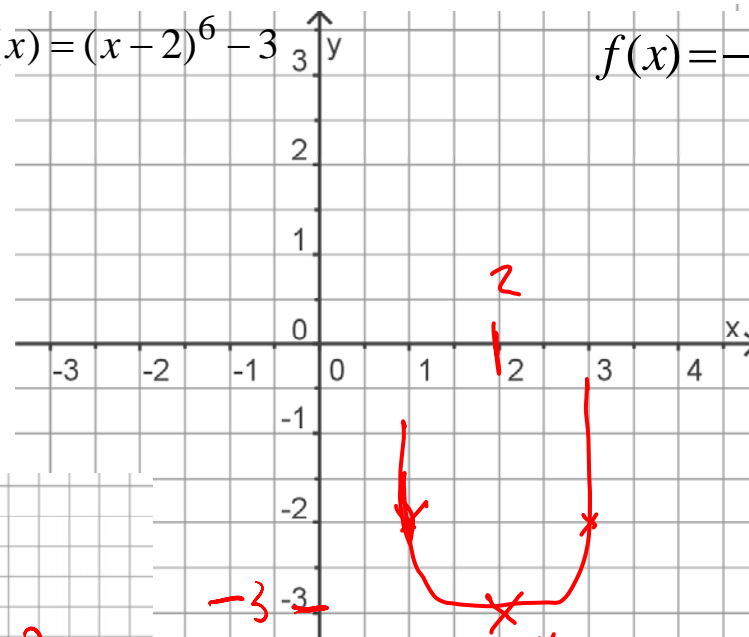
do it yourself

exercises with power functions

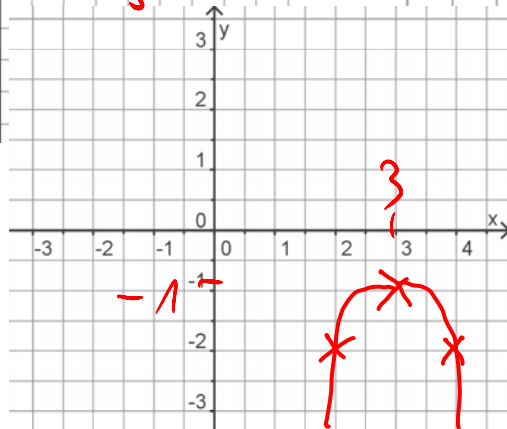
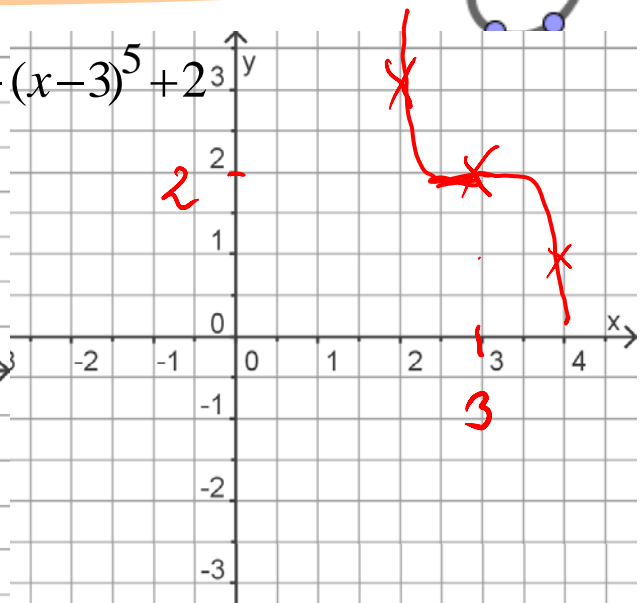
$$f(x) = (x - 3)^4 + 1$$



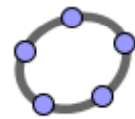
$$f(x) = (x - 2)^6 - 3$$



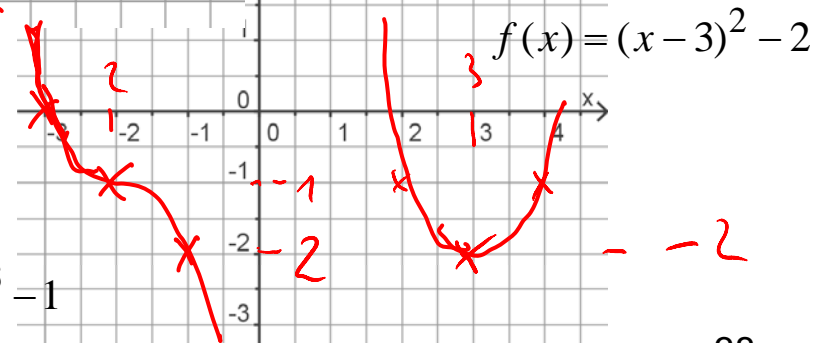
$$f(x) = -(x - 3)^5 + 2$$



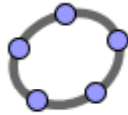
$$f(x) = -(x - 3)^4 - 1$$



$$f(x) = -(x + 2)^3 - 1$$



Parabeln



Parabolas

