

4. Modelling, Markov chains [10 points]

Transition probabilities in %	citizens	free farmers	bondsmen
citizens	80	0	20
free farmers	20	40	40
bondsmen	40	10	50

$$A^{50} = \begin{pmatrix} 0,65 & 0,05 & 0,3 \\ 0,65 & 0,05 & 0,3 \\ 0,65 & 0,05 & 0,3 \end{pmatrix}$$

In medieval times the free farmers became more and more dependent upon the landlords. They could only escape by hiding out in a town for one year. One said "Stadtluft macht frei." Prof. Mediavi researched the migratory movements of citizens, free farmers and bondsmen in Schwaben, a region in southern Germany. The transition matrix of his model is shown on the top left. The described period of **one** calculation is one decade.

4.1 a) Draw the state graph.

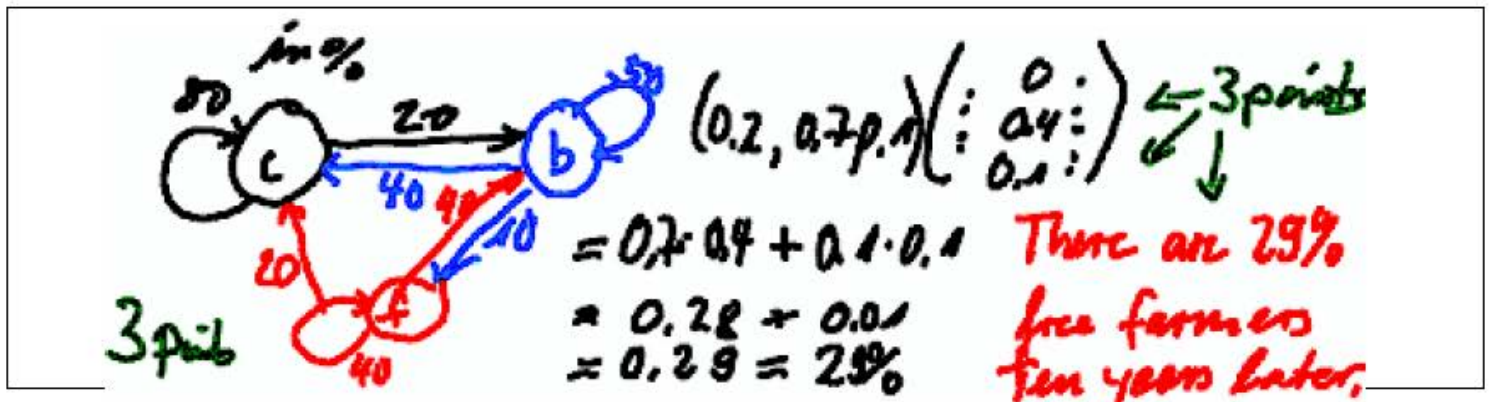
b) When this trend startet in the Weser-mountains, a region in northern Germany, 20% citizens, 70% free farmers and 10% bondsmen lived there. What percentage of free farmers will be there after the next decade in the Weser-mountains according to Prof. Mediavi's model? (approach, traceable calculation, answer in a sentence)

Nur vom Prüfer auszufüllen

6P

	10	20	30	40	50	60	70	80	90	
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	0	1	2	3	4	5	6	7	8	9

+ 0,5



- MC 4.2 **mc-question:** Which of the following statements are correct as an interpretation of the A^{50} (see top right)?
- 1.0P The free farmers lost their livelihood due to bad harvests and higher tributes to the landlords.
 - 2.0P In the long term there is a 65% probability that only citizens will remain, a 5% probability that only free farmers will remain and a 30% probability that only bondsmen will remain.
 - 2.0P In the long term, due to this model, 65% of the population will live as citizens, 5% as free farmers and 30% as bondsmen.
- MC 4.3 **mc-question:** Prof. Mediavi, see task at the top, is a good historian. As a competent scientist, what would be his reaction if finding a source which shows that a century later there were mostly the same fractions of citizens, free farmers and bondsmen in the Weser-Mountains.
- 2.0P He keep silent about the Weser-mountains and looks for another region which is closer to his model.
 - 1.0P He researches the reasons why the free farmers in the Weser-mountains stayed their ground better than in Schwaben.
 - 1.0P He extends and modifies his model so that he can take account of even more factors.